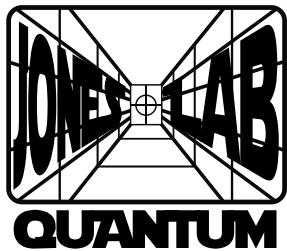


# Using Co-Designed Architectures for Modular Superconducting Quantum Computers

Evan McKinney<sup>†</sup>,

M. Xia<sup>§</sup>, C. Zhou<sup>§</sup>, P. Liu<sup>§</sup>, M. Hatridge<sup>§</sup>, A.K. Jones<sup>†</sup>



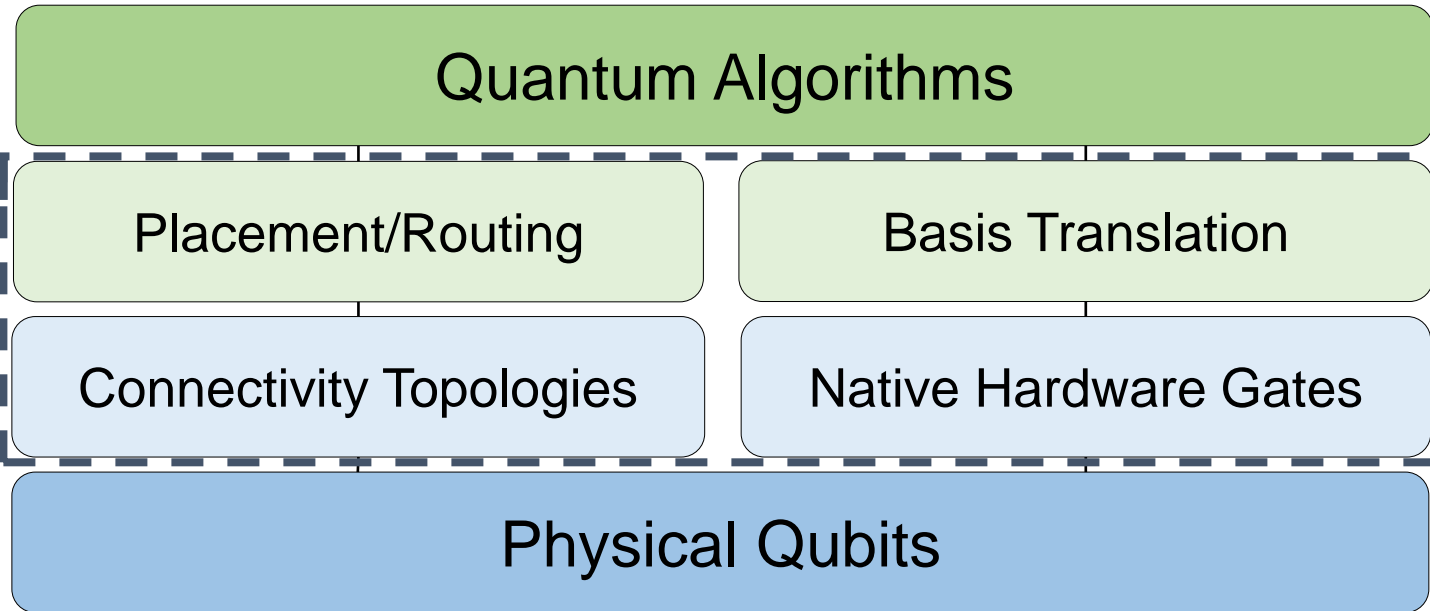
<sup>†</sup>Department of Electrical and Computer Engineering, University of Pittsburgh

<sup>§</sup>Department of Physics and Astronomy, University of Pittsburgh

HPCA, 2023



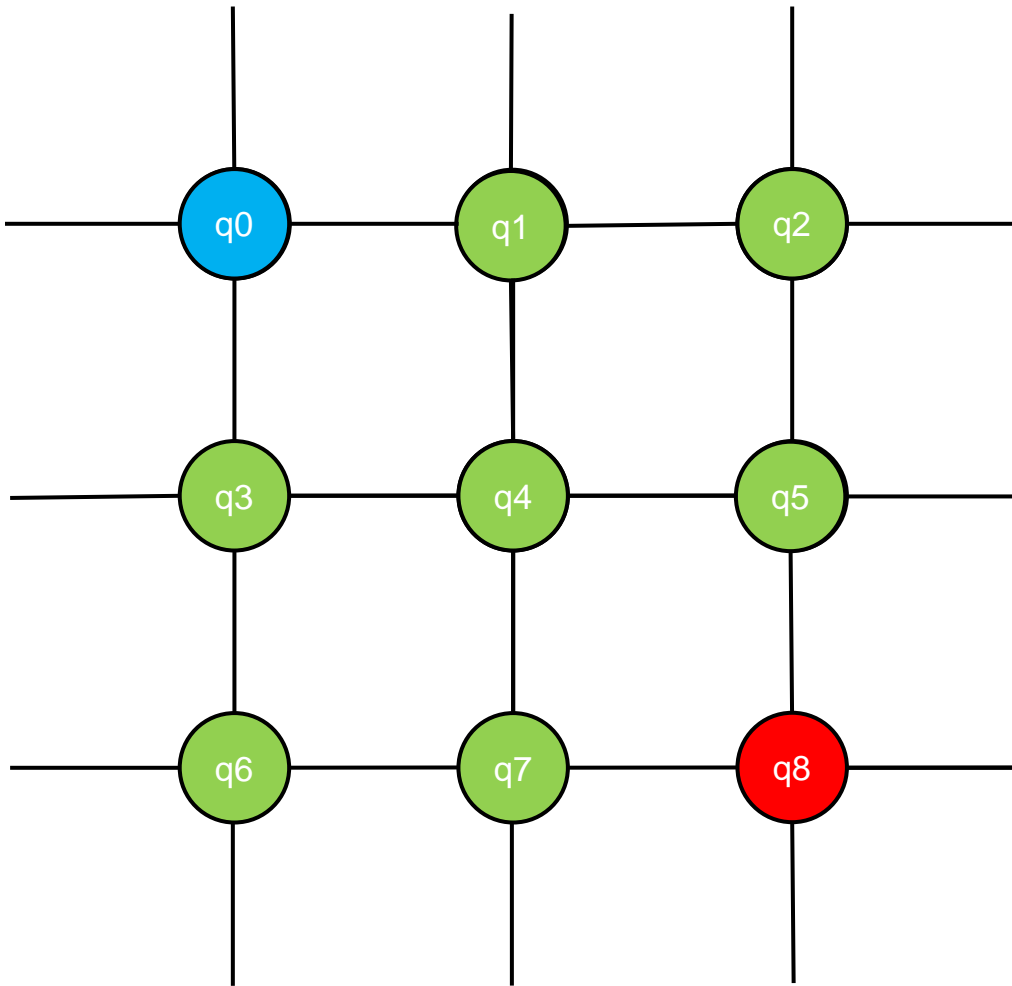
Transpilation ←



- Physics constrains possible topologies and basis gates
- Prioritize improving qubit and gate fidelities

# Qubit routing with SWAPs

## ➤ Example: Square-Lattice Topology

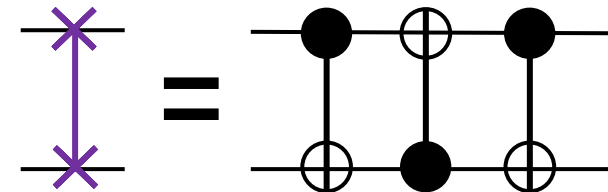


## ➤ Inducing SWAP gates on a circuit

SWAP-minimization is NP-complete

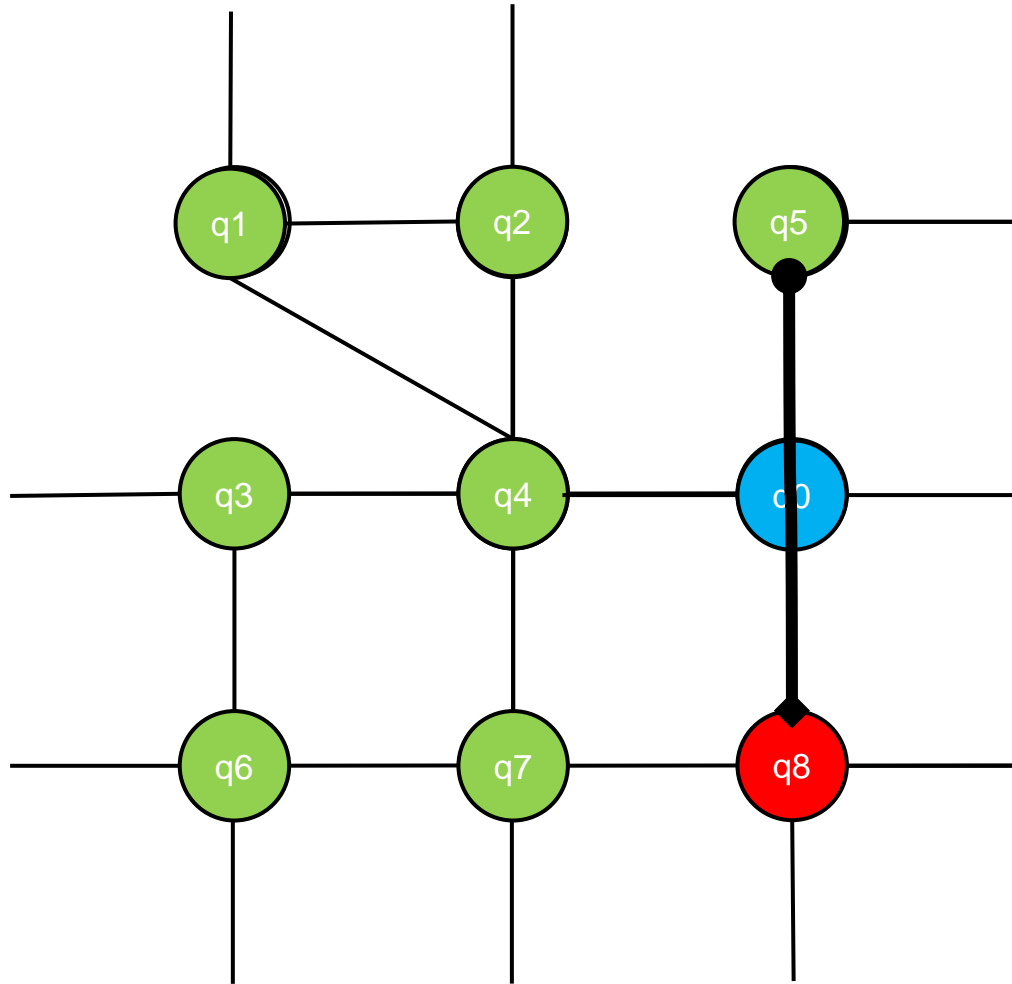


## ➤ **SWAPs are expensive**, we desire connectivities which minimize the need for data movement



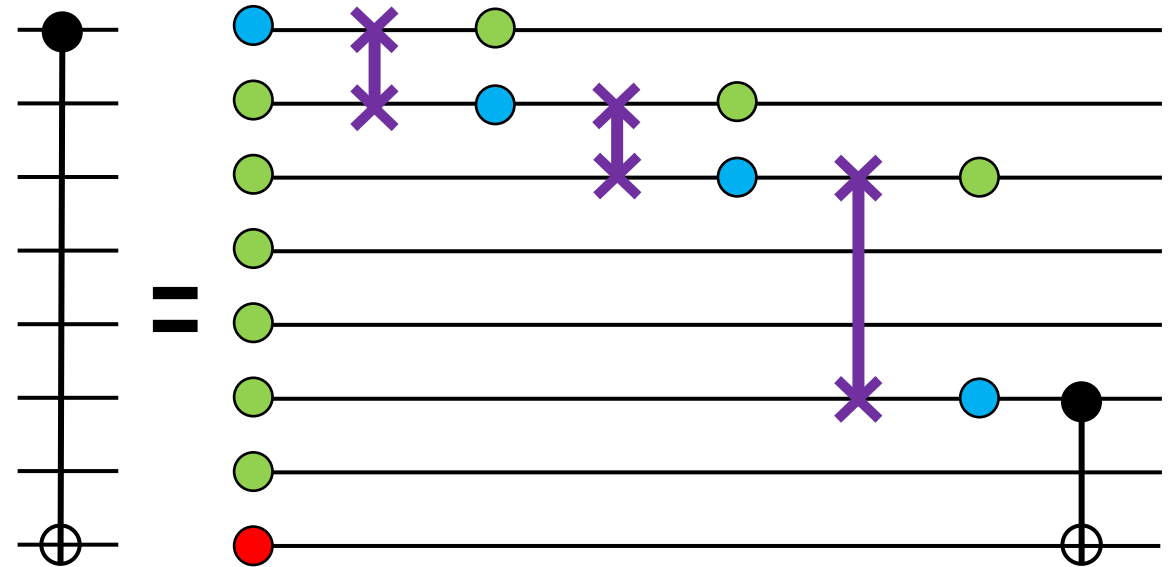
# Qubit routing with SWAPs

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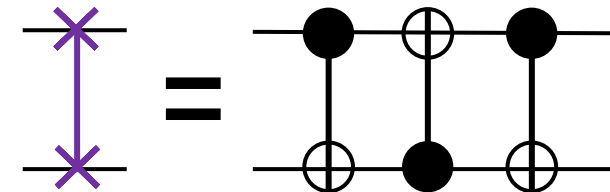


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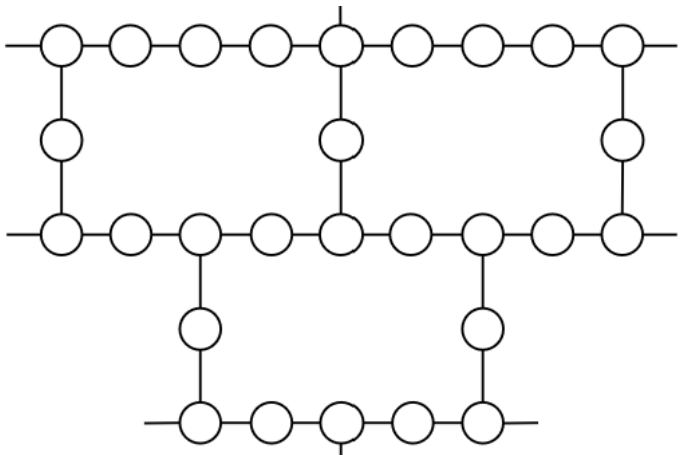


# Motivating co-design

The choice of gate type and coupling topology are not independent, as they are both determined by the choice of *modulator*.

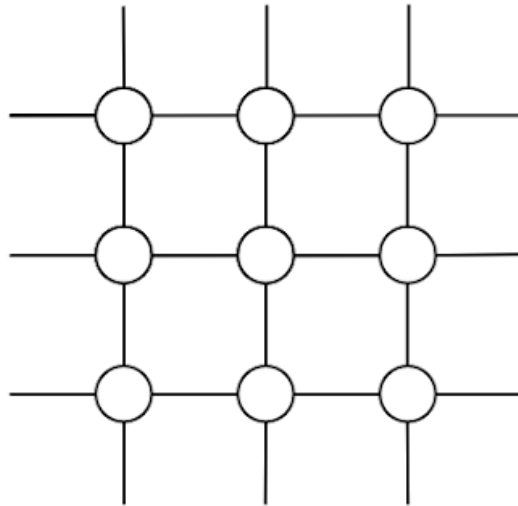
## IBM

$$ZX(\theta) = \begin{bmatrix} \cos \theta/2 & 0 & -i \sin \theta/2 & 0 \\ 0 & \cos \theta/2 & 0 & i \sin \theta/2 \\ -i \sin \theta/2 & 0 & \cos \theta/2 & 0 \\ 0 & i \sin \theta/2 & 0 & \cos \theta/2 \end{bmatrix}$$



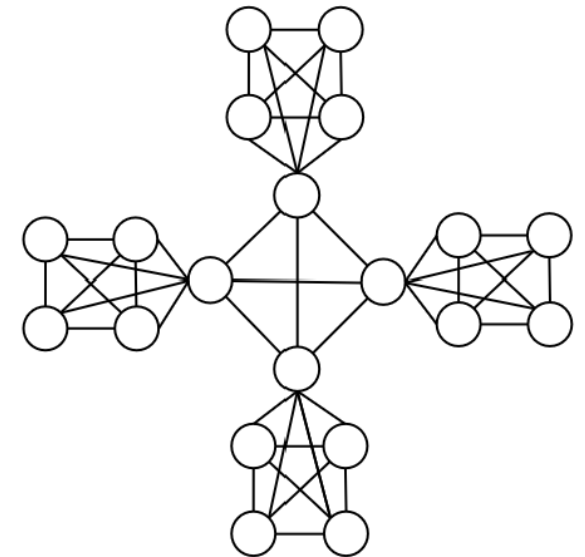
## Google

$$FSIM(\theta, \phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -i \sin \theta & 0 \\ 0 & -i \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{bmatrix}$$



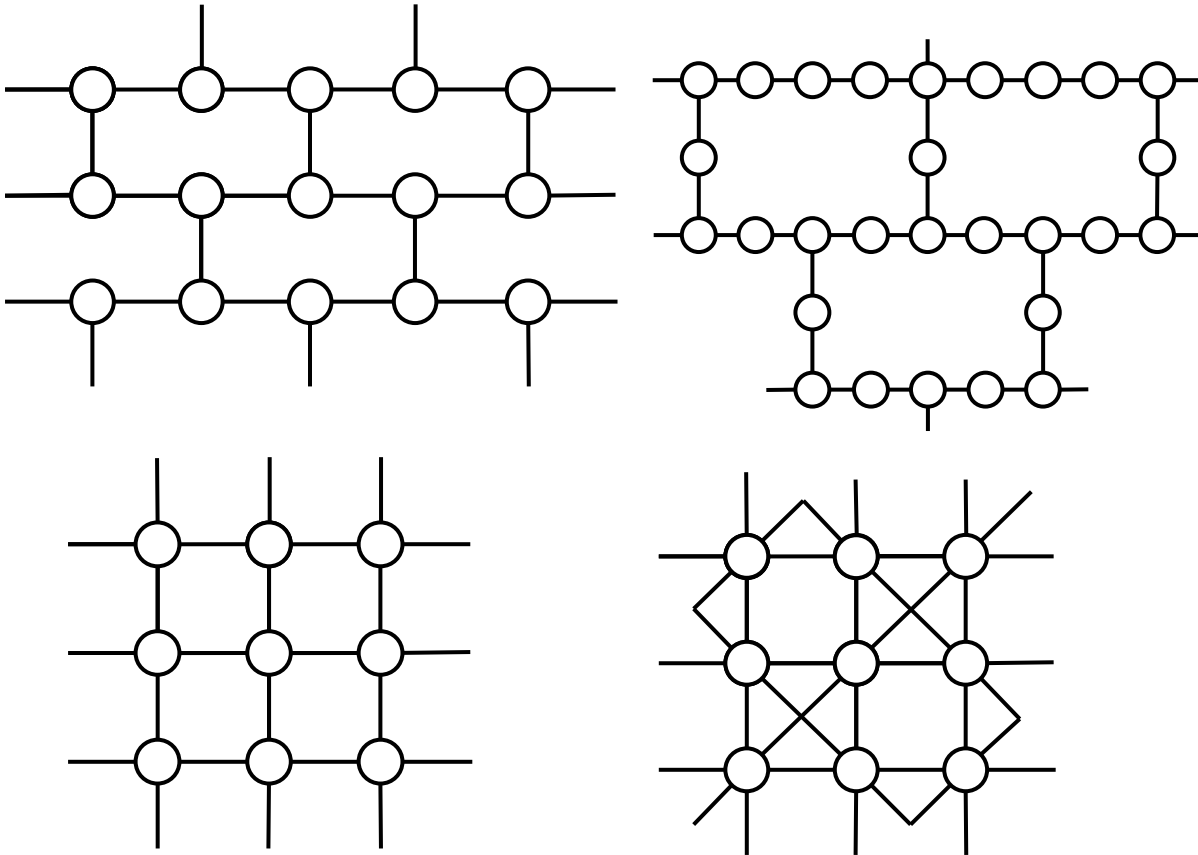
## Hatlab

$$\sqrt[n]{i\text{SWAP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi/2n) & i \sin(\pi/2n) & 0 \\ 0 & i \sin(\pi/2n) & \cos(\pi/2n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Coupling topologies

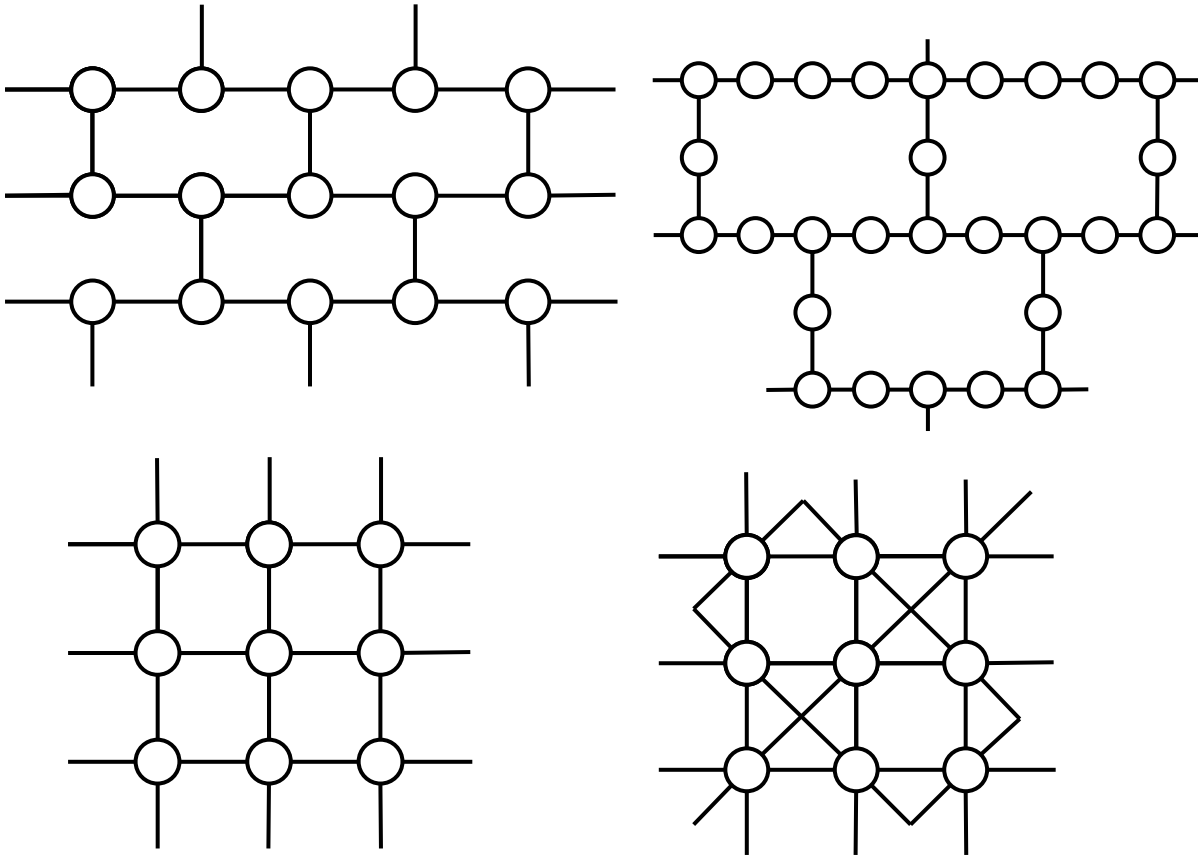
## ➤ Common Planar Topologies [1,2]



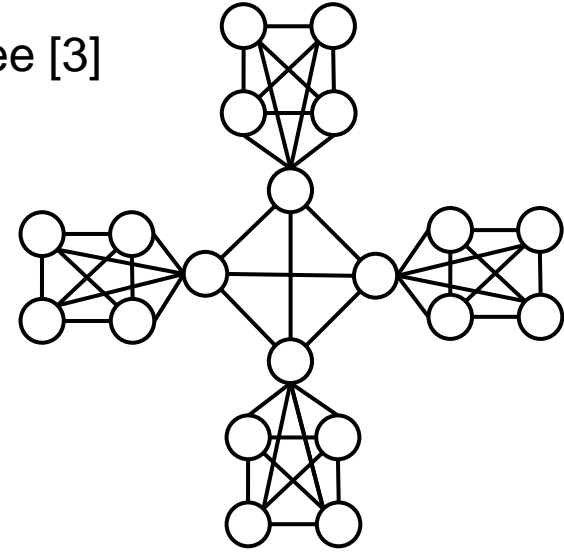
- [1] Nation, et al. **IBM** (2021)
- [2] Arute, et al. **Nature** (2019)
- [3] Zhou, et al. **arXiv**: 2109.06848 (2021)

# Coupling topologies

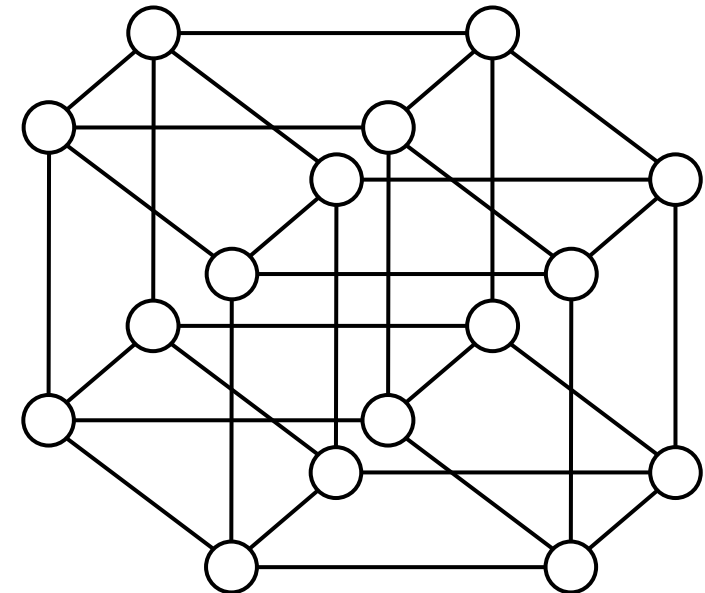
## ➤ Common Planar Topologies [1,2]



## ➤ Modular Tree [3]



## ➤ 4D-Hypercube



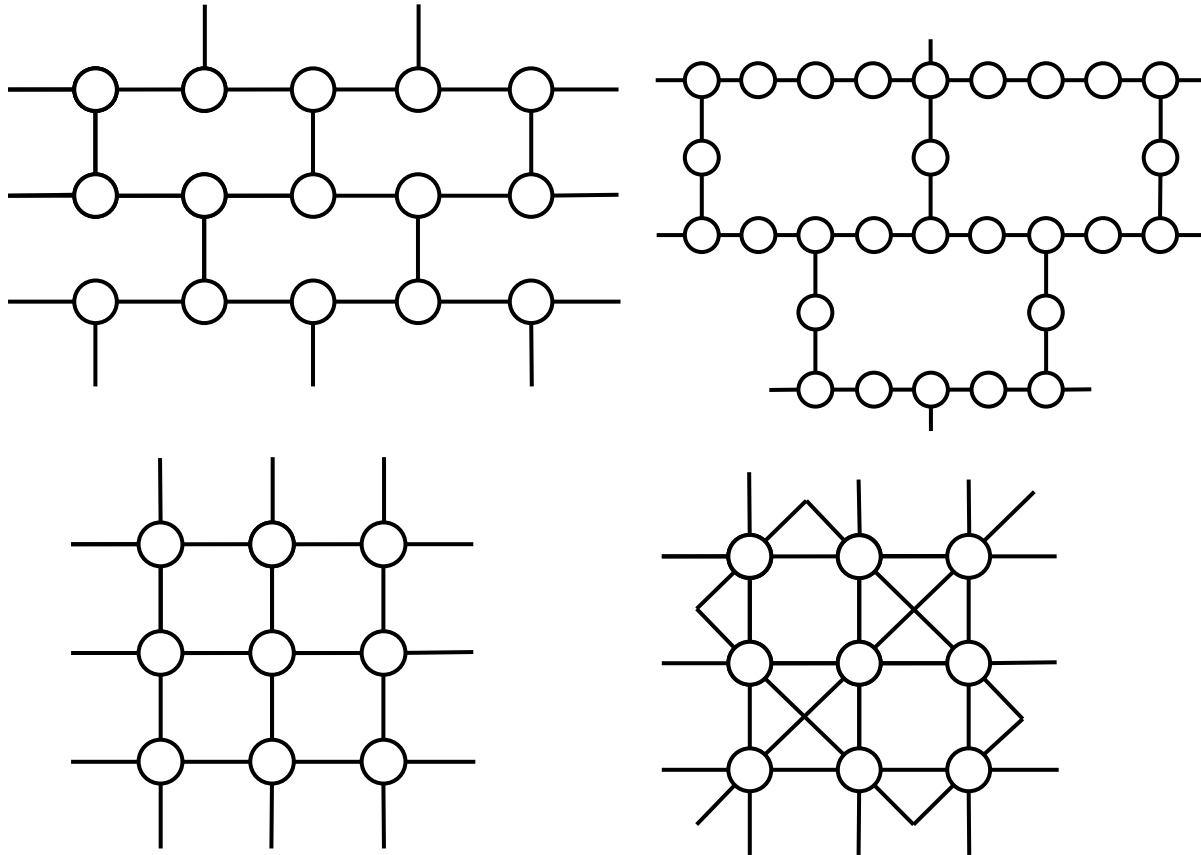
[1] Nation, et al. **IBM** (2021)

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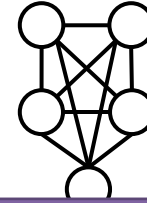
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# Coupling topologies

## ➤ Common Planar Topologies [1,2]

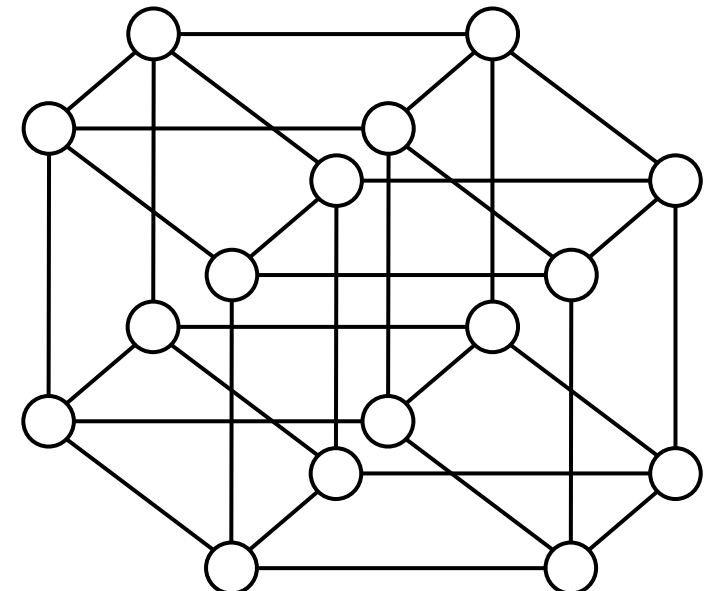


## ➤ Modular Tree [3]



Useful in classical networking but is difficult to physically realize

## ➤ 4D-Hypercube

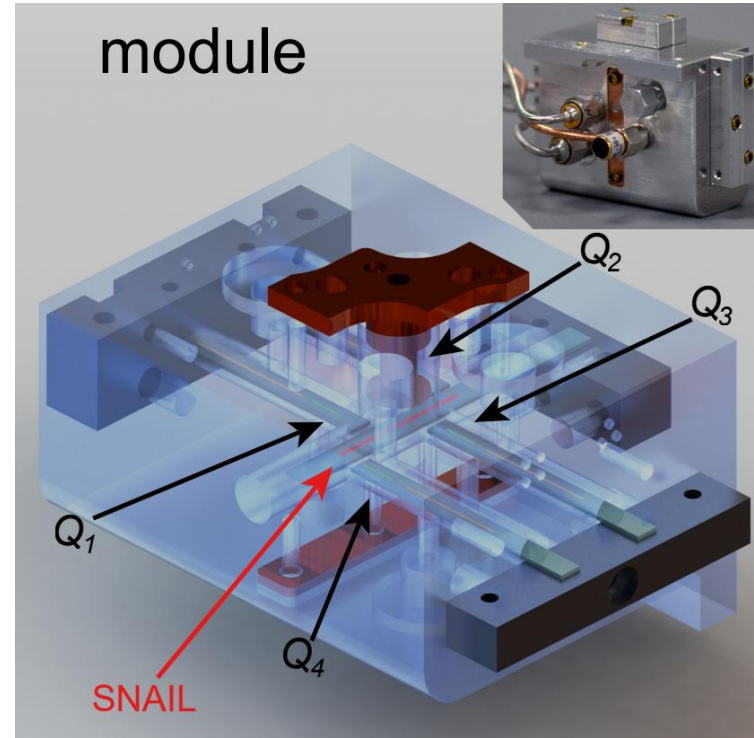
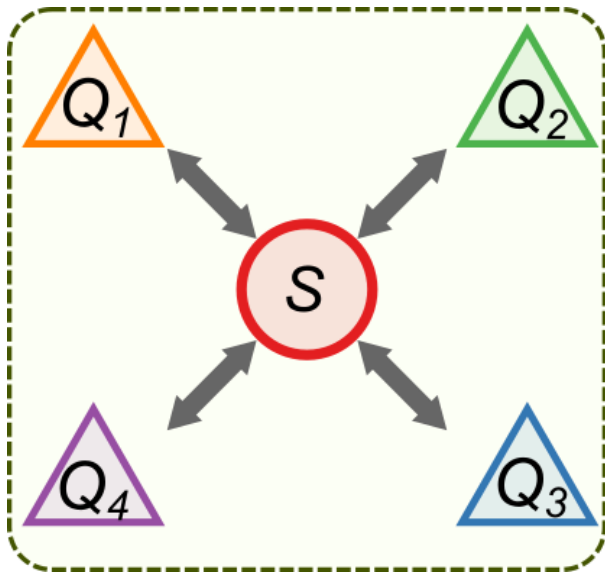


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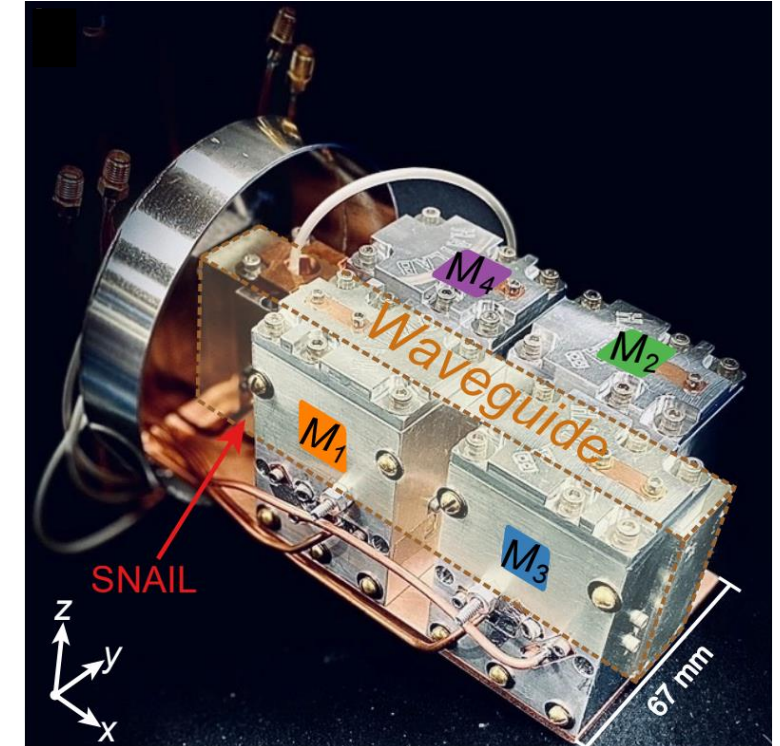


## Four qubit SNAIL-based quantum module

### Module



Rendered module and image

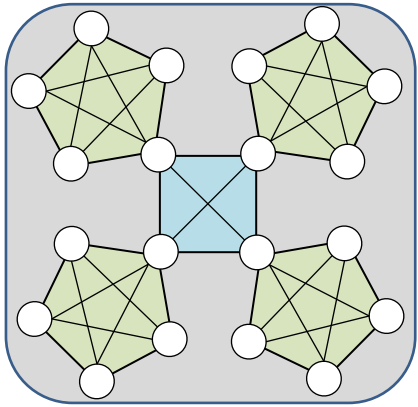


Assembled device

# Scaling SNAIL tree topologies

20 Qubit Trees

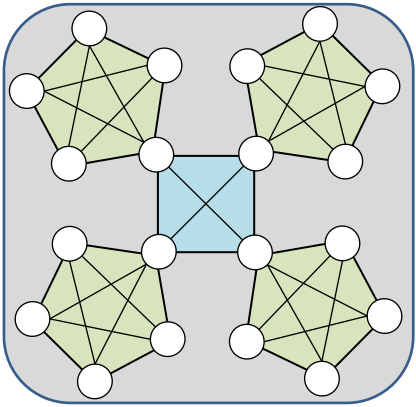
**4-ary**



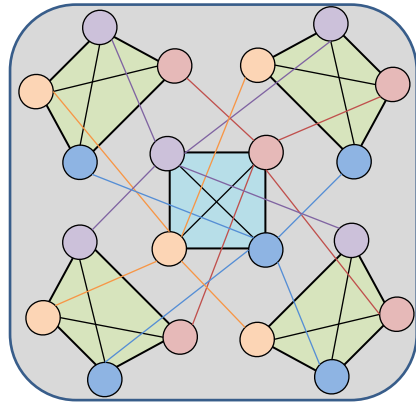
# Scaling SNAIL tree topologies

20 Qubit Trees

**4-ary**



**interleaved**

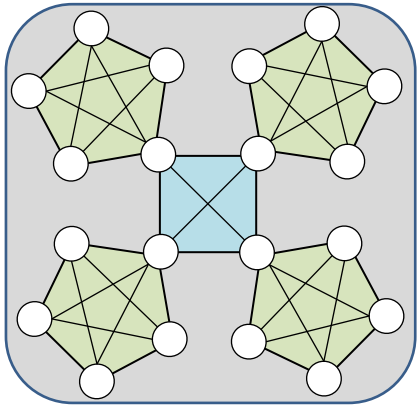


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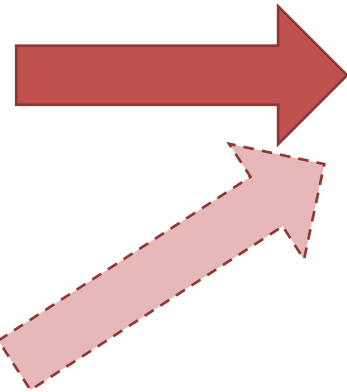
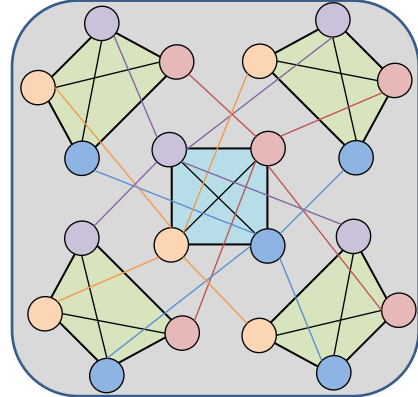
20 Qubit Trees

84 Qubit Tree

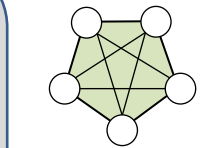
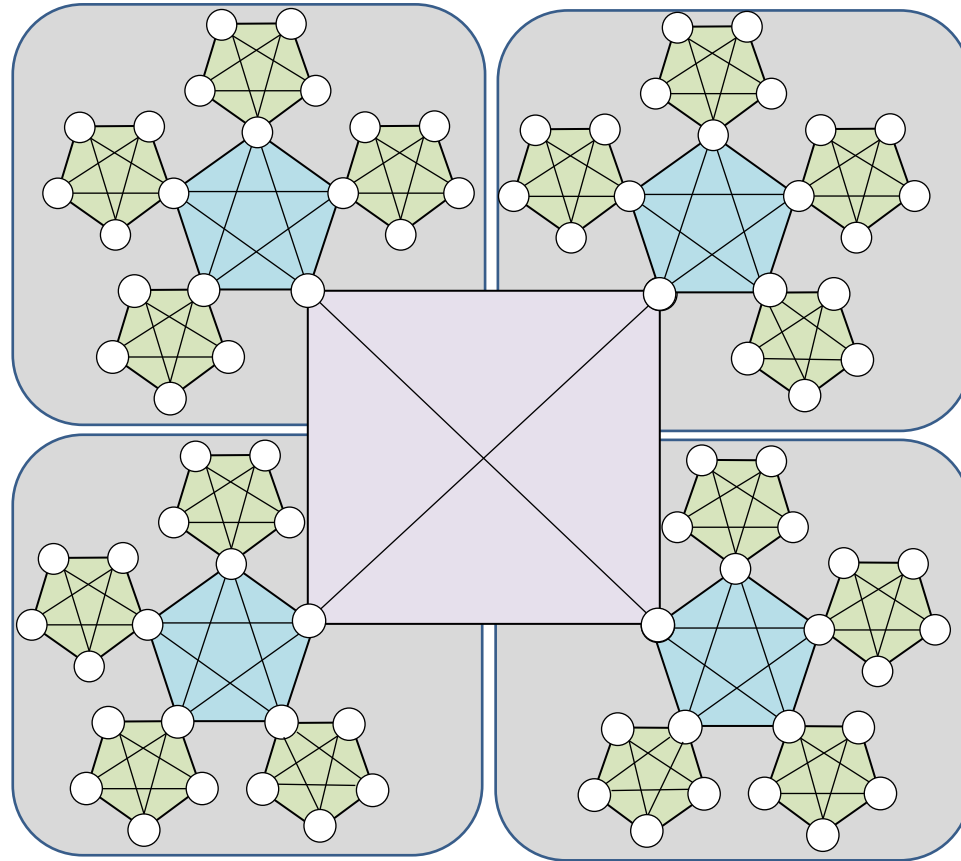
**4-ary**



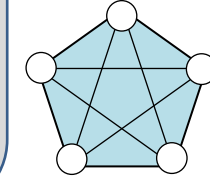
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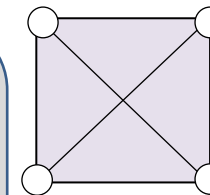
**5-ary**



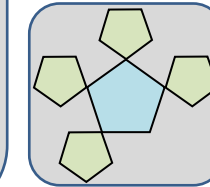
Level 1 Module



Level 2 Module



Level 3 Module

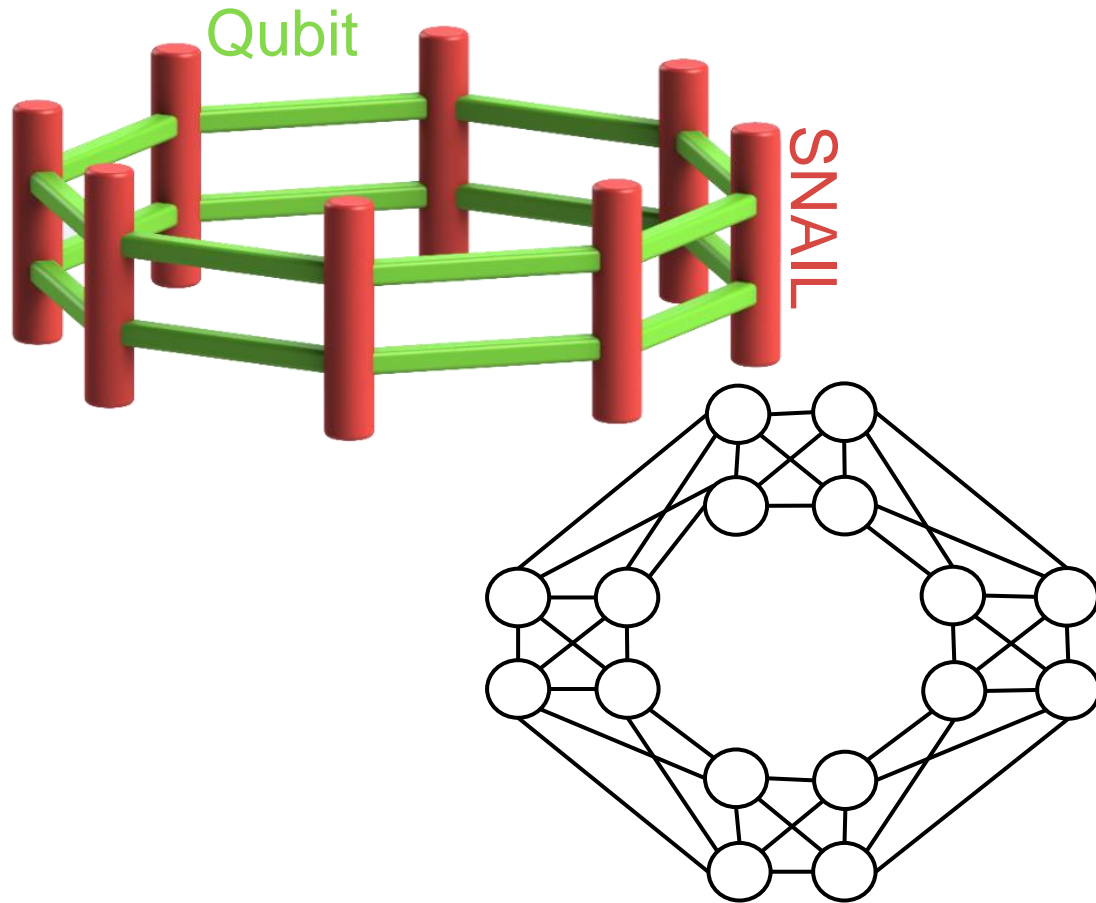


Levels 1&2

# Co-designed SNAIL topologies

**Objective:** Maintain the low- diameter property of hypercubes without the poor dimensionality scaling.

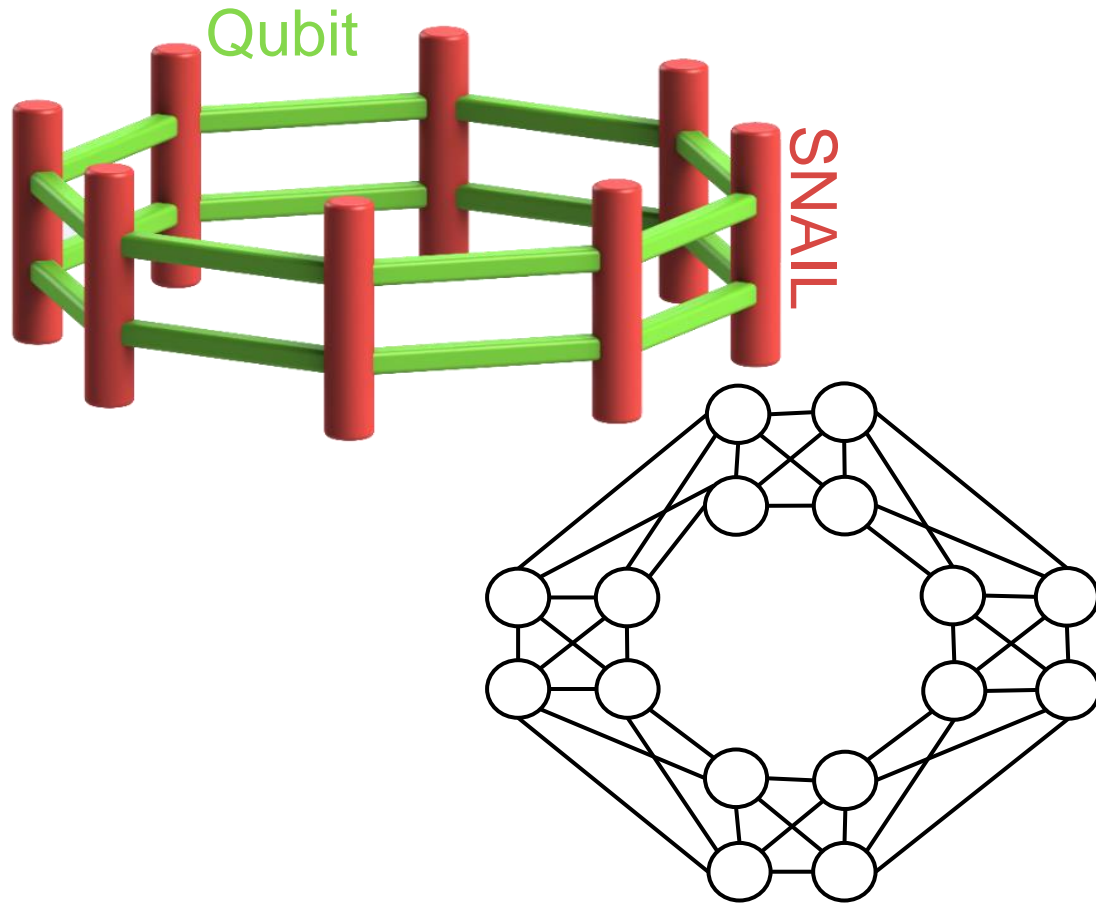
16 Qubit Corral<sub>11</sub>



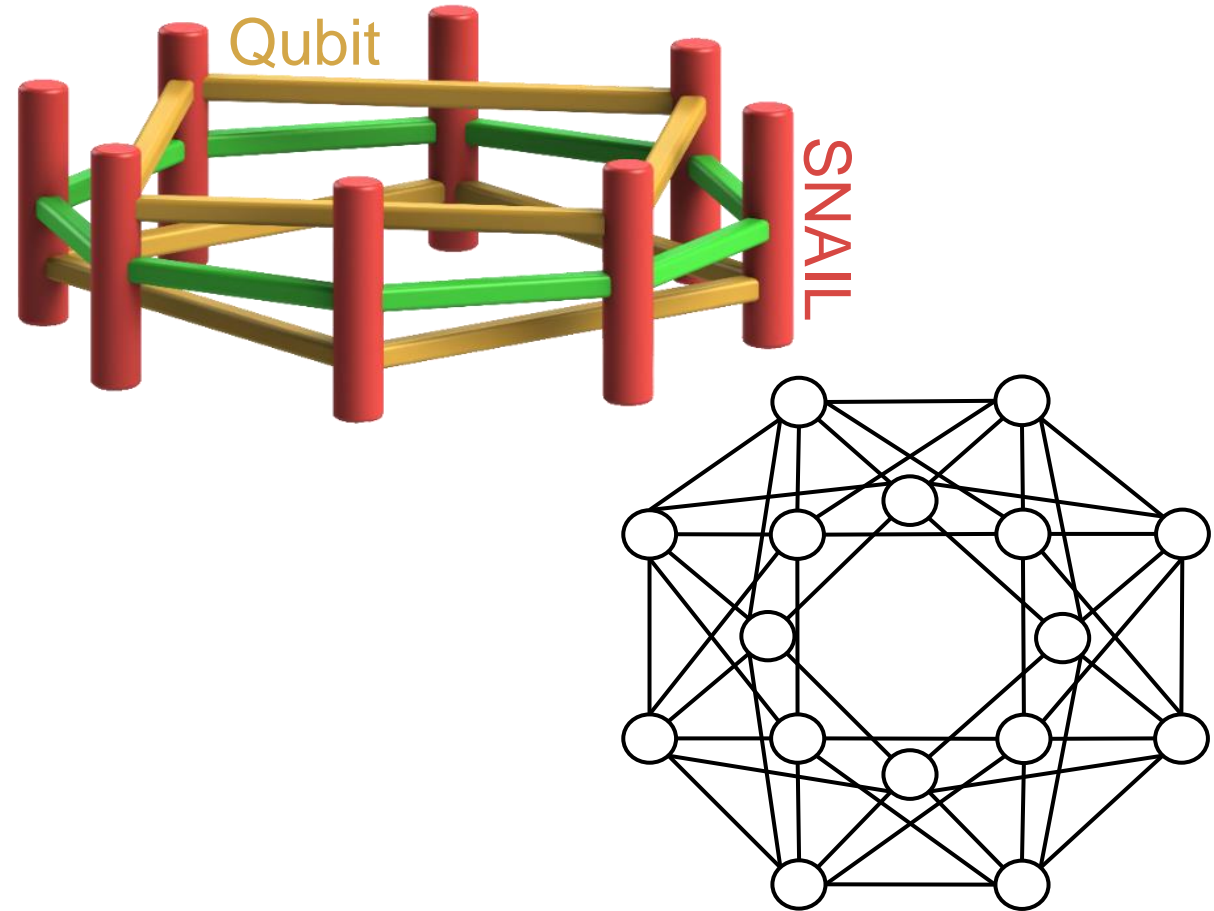
# Co-designed SNAIL topologies

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16 Qubit Corral<sub>11</sub>



16 Qubit Corral<sub>12</sub>

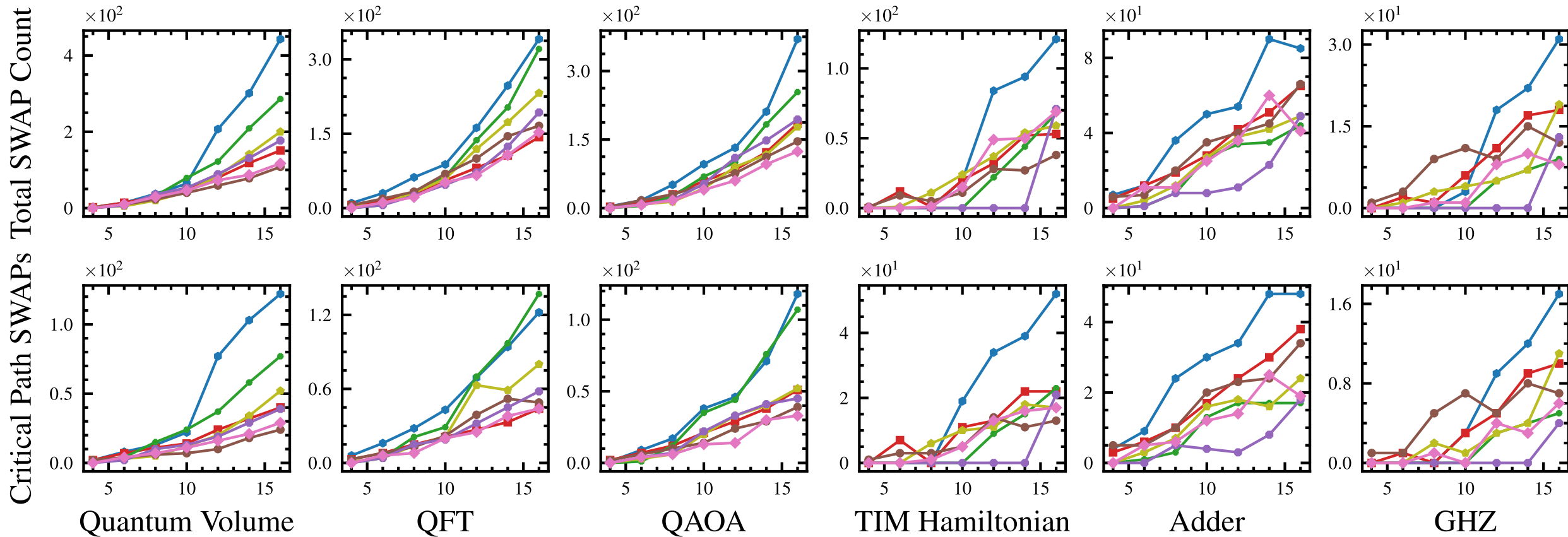


# Benchmarking small topologies

◆ Heavy-Hex  
■ Square-Lattice

★ Tree  
◆ Tree-I

● Hypercube  
● Corral<sub>1,1</sub>  
◆ Corral<sub>1,2</sub>

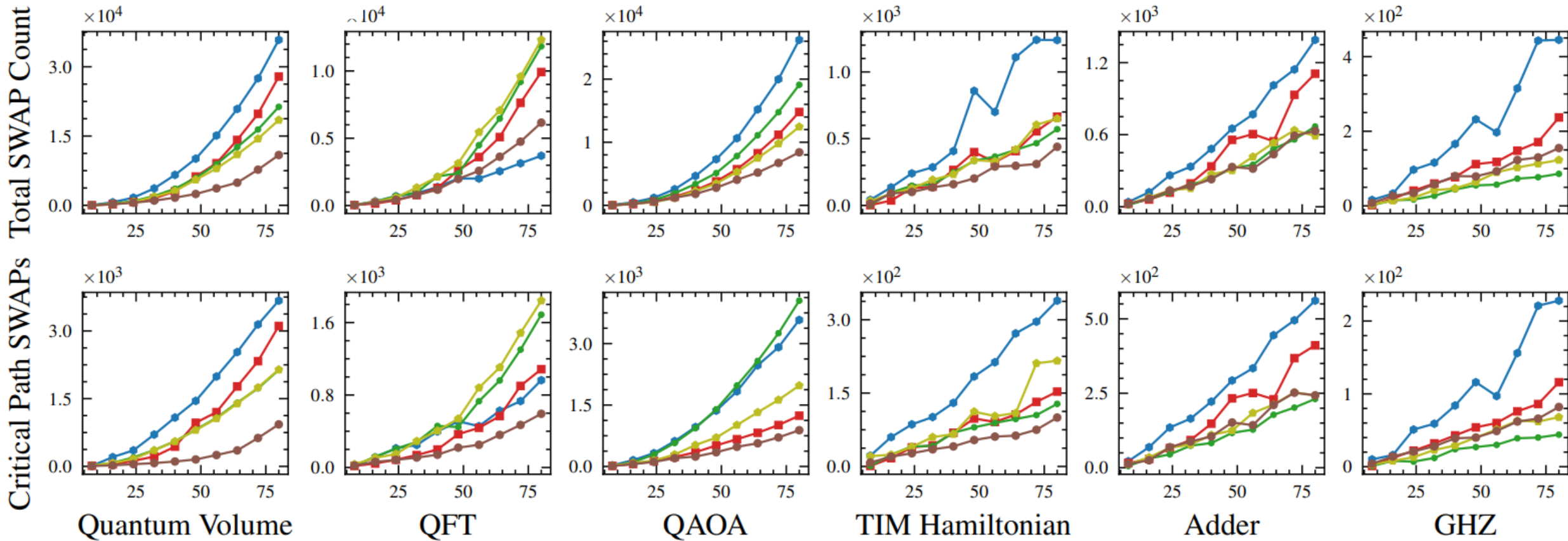


- **Even smaller 16Q “neighborhoods” can benefit**
  - Heavy-Hex is 82% slower (<< fidelity) than Corral<sub>1,1</sub>

◆ Heavy-Hex  
■ Square-Lattice

# Benchmarking large topologies

★ Tree  
◆ Tree-1  
● Hypercube

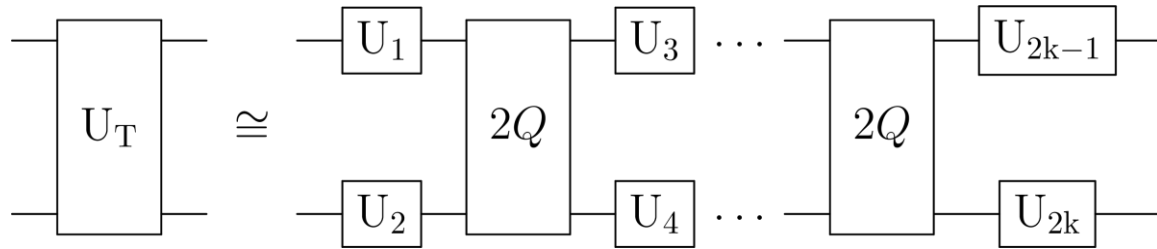


- **Sparse topologies require more SWAP gates when scaled**
  - 80Q Heavy-Hex induces 3X critical path SWAPs vs. Hypercubes



# Two-qubit basis gates

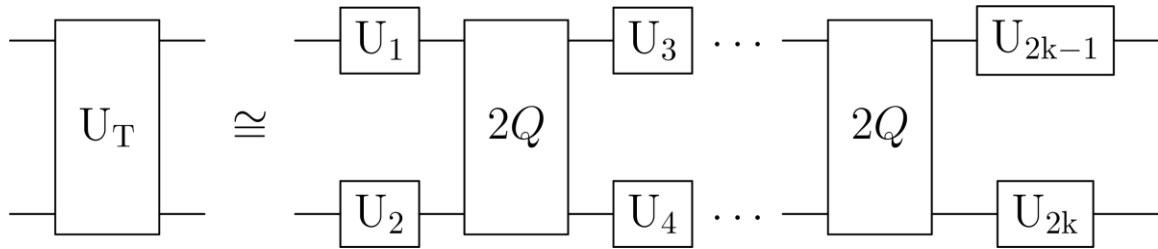
- Decompose all algorithm gates into new basis using repeated applications



- An optimal basis gate *reduces overall duration*
  - Powerful gates need less applications
  - Fidelity limited by decoherence in time

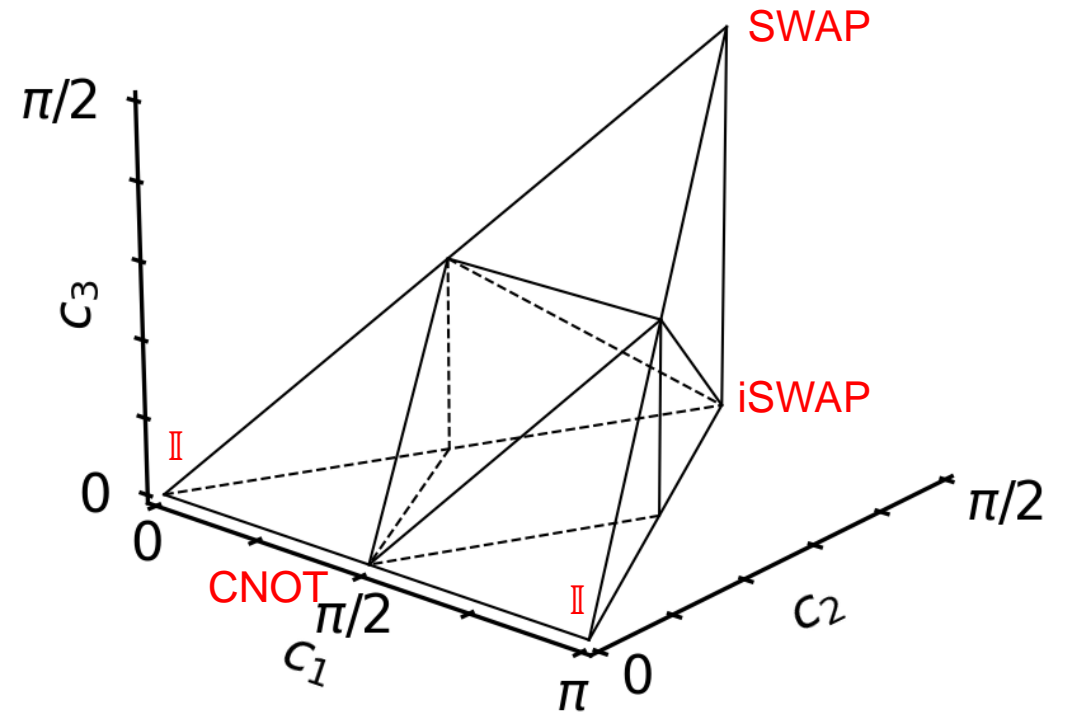
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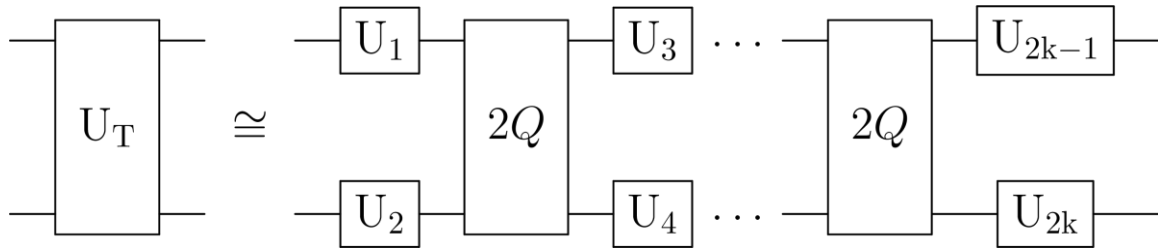
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- Weyl Chamber visualizes the set of all 2Q gates



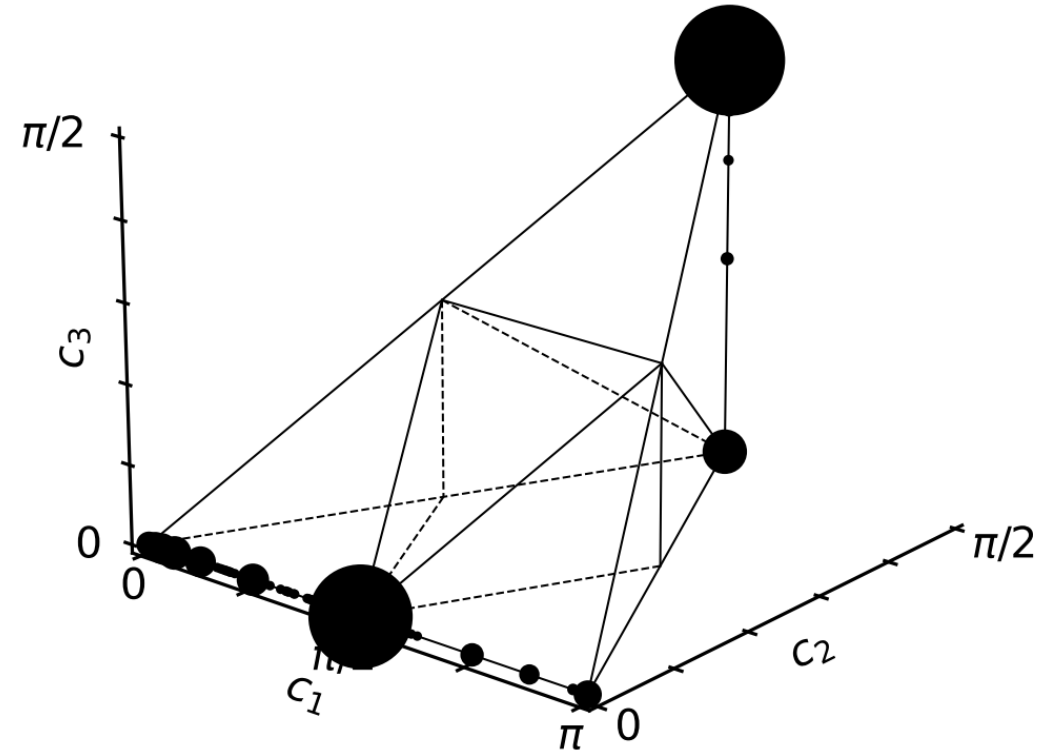
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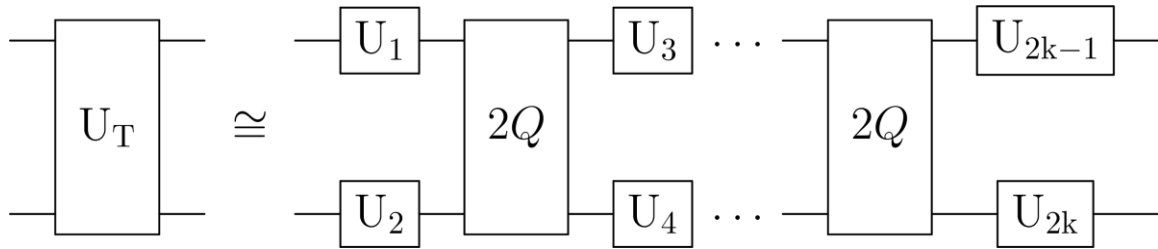
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- NISQ algorithms dominated by CX and SWAP gates

# Two-qubit basis gates

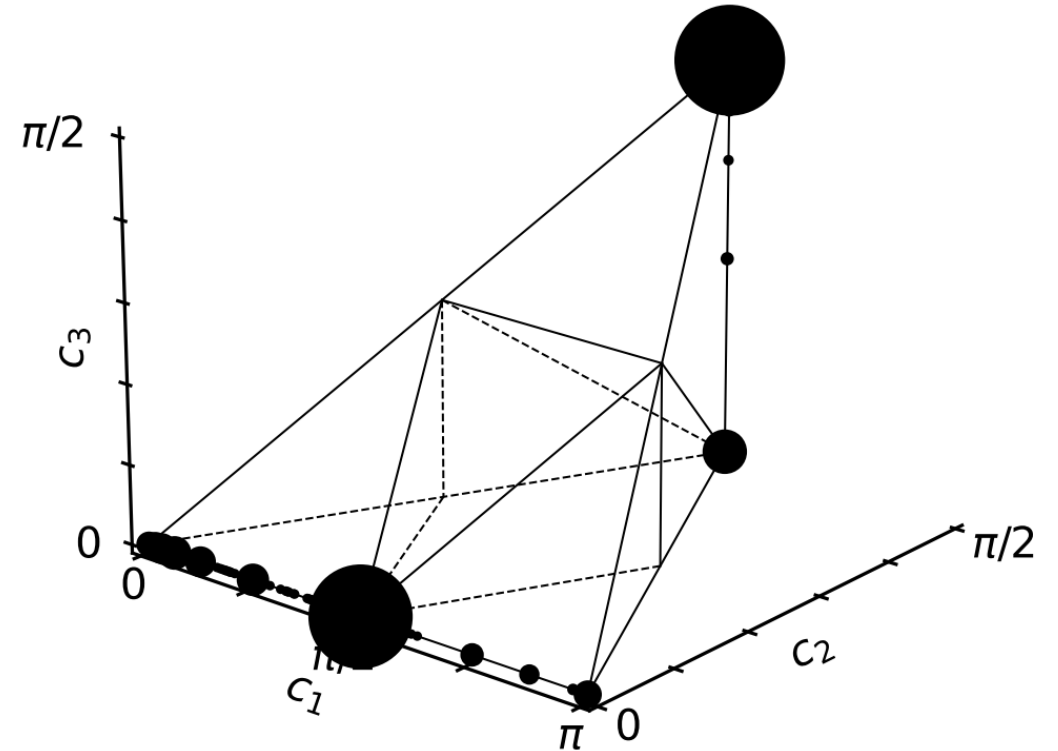
- Decompose all algorithm gates into new basis using repeated applications



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  - Fidelity limited by decoherence in time

- **Goal: Use both decomposition efficiency and hardware latency = overall duration**

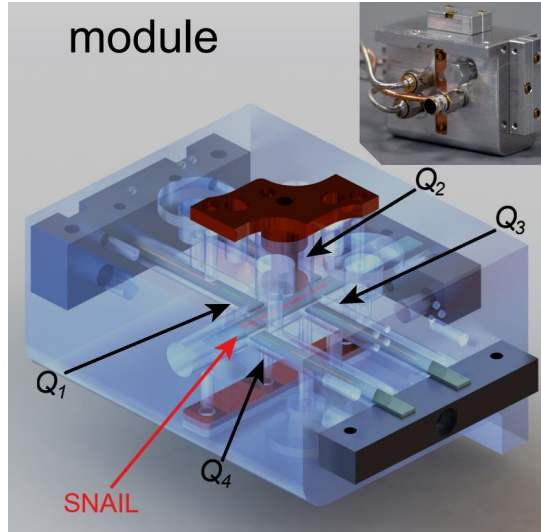
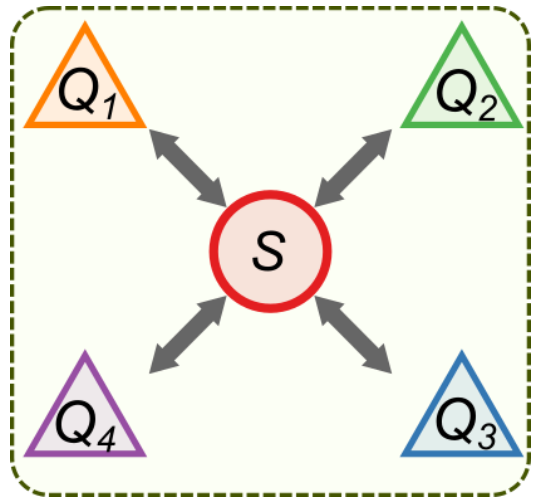
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# Conversion/Gain candidate basis gates

Four qubit SNAIL-based quantum module

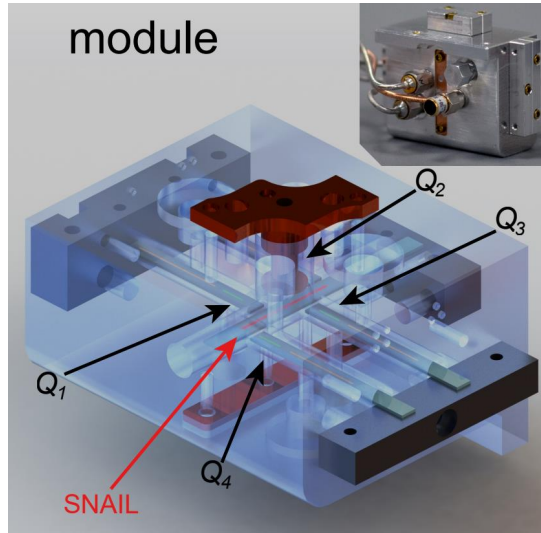
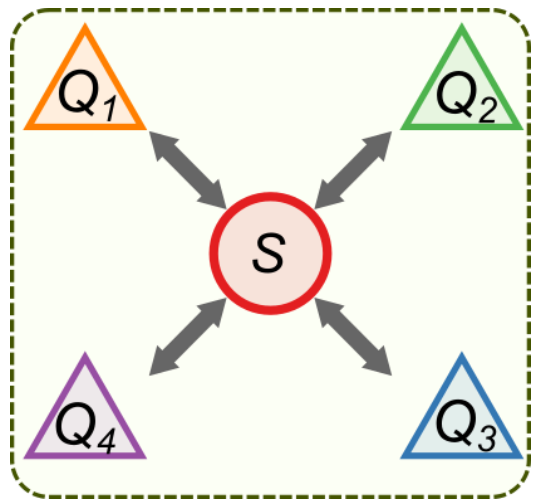


➤ Engineerable interactions yields a basis gate design-space

$$\hat{H} = g_c(e^{i\phi_c} a^\dagger b + e^{-i\phi_c} ab^\dagger) + g_g(e^{i\phi_g} ab + e^{-i\phi_g} a^\dagger b^\dagger)$$

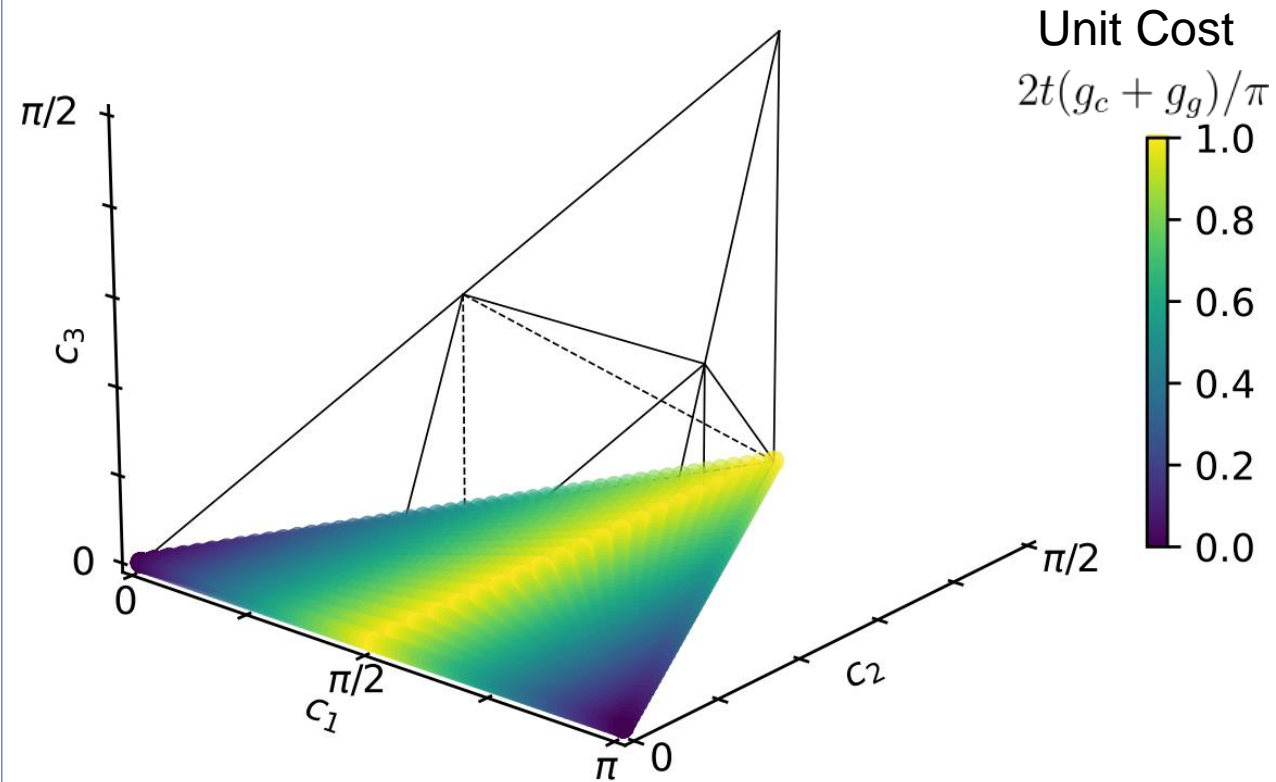
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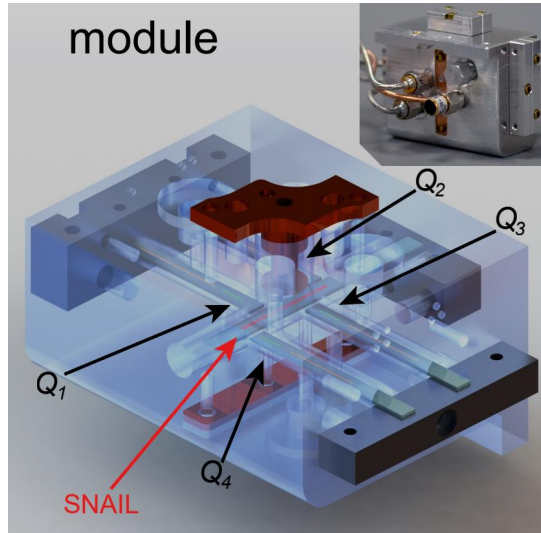
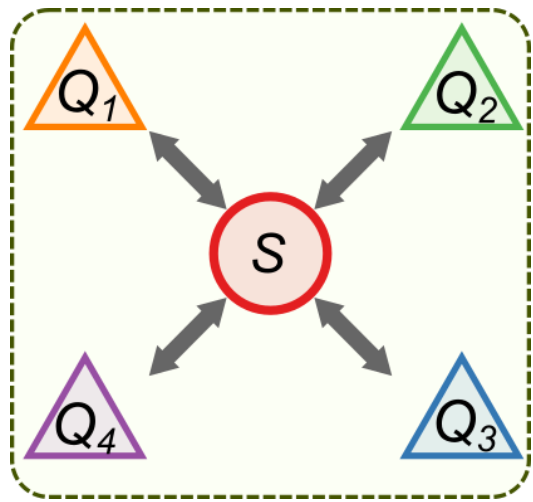
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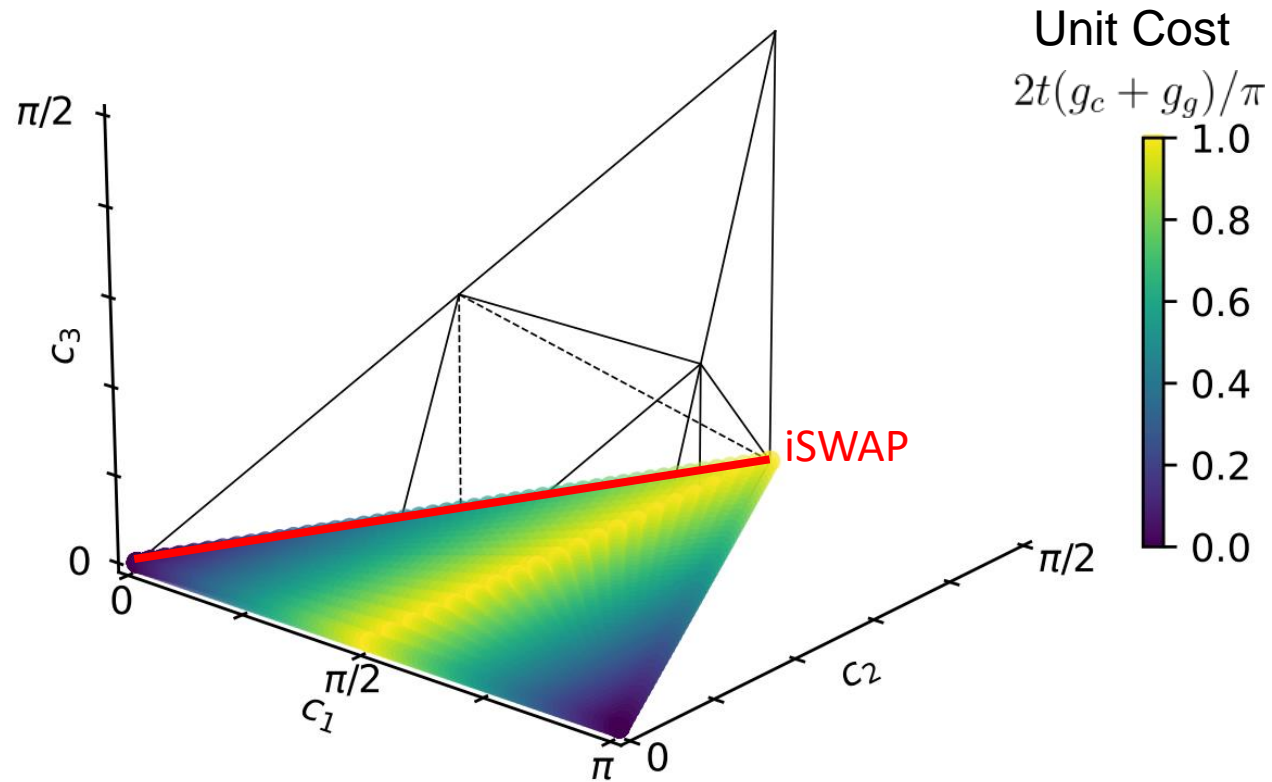
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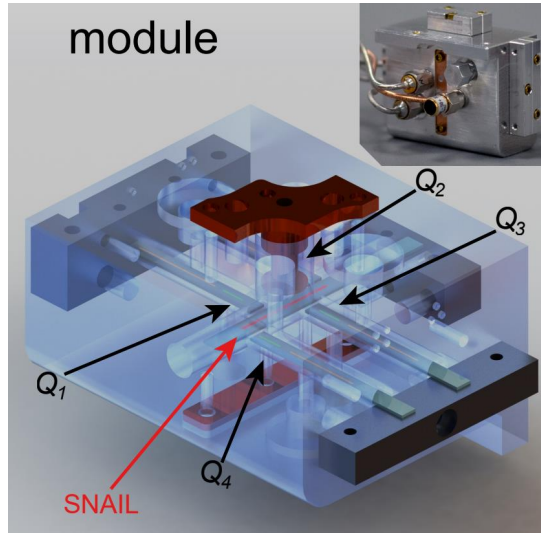
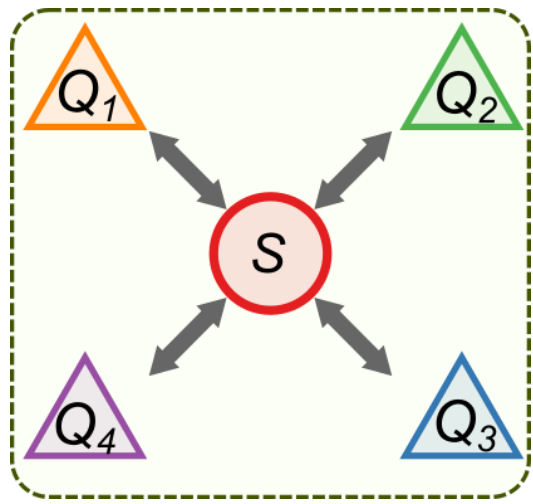
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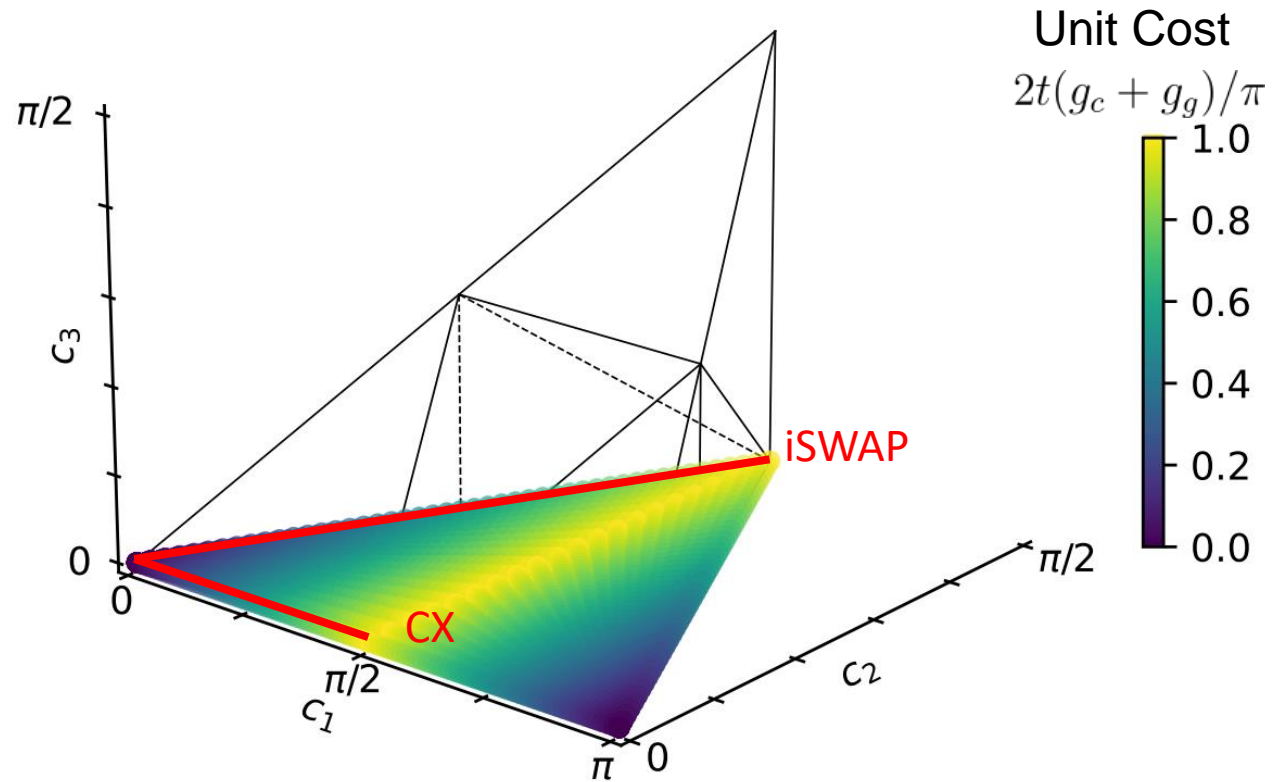
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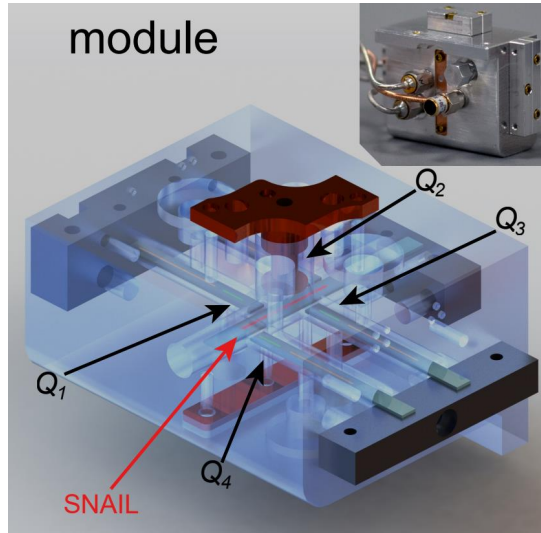
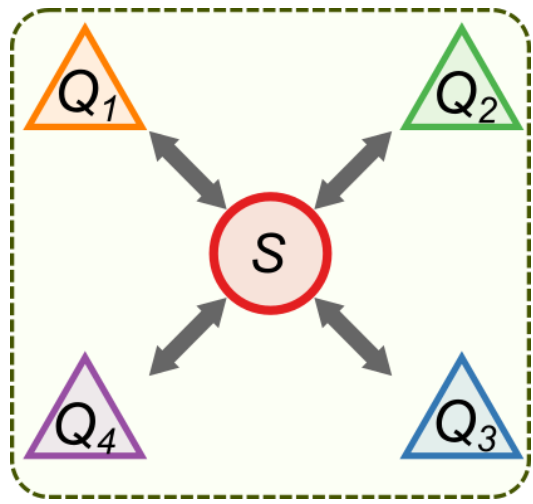
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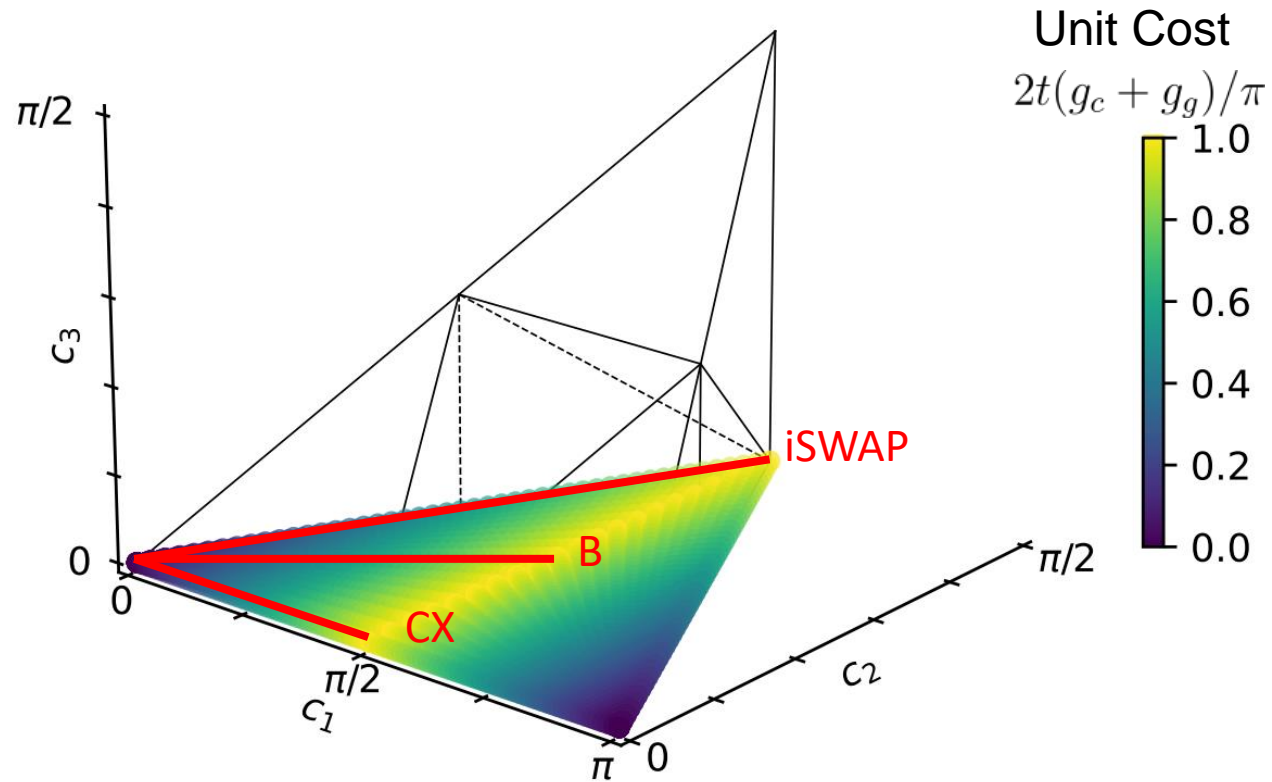
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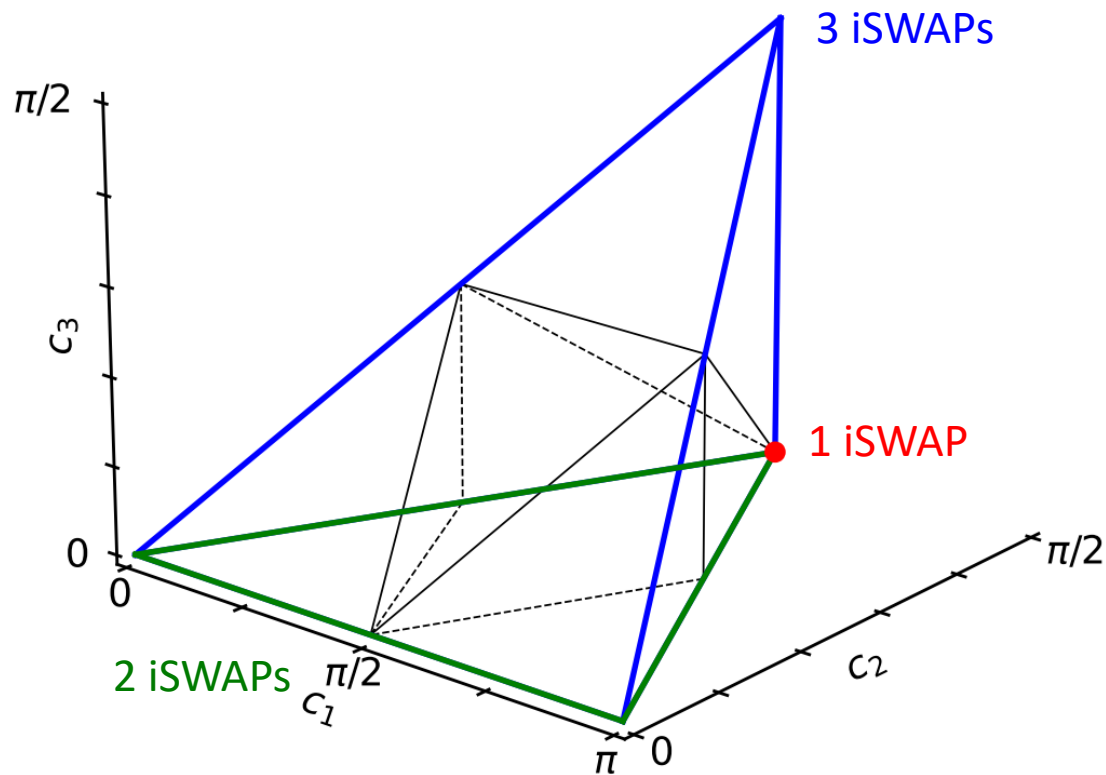


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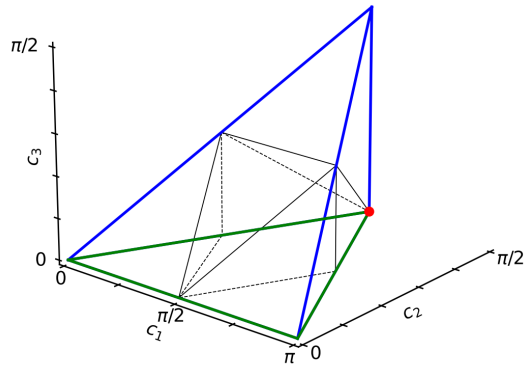
# Basis coverage volumes



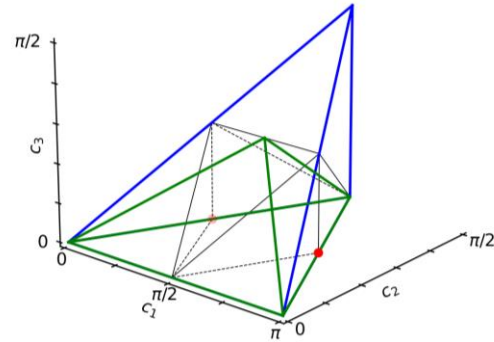
- **Monodromy polytopes** finds minimum gate applications for any 2Q target gate
- A single gate locally equivalent to itself
- SWAP is the most expensive target

Basis	iSWAP
CNOT	2.0
SWAP	3.0
Haar	3.0

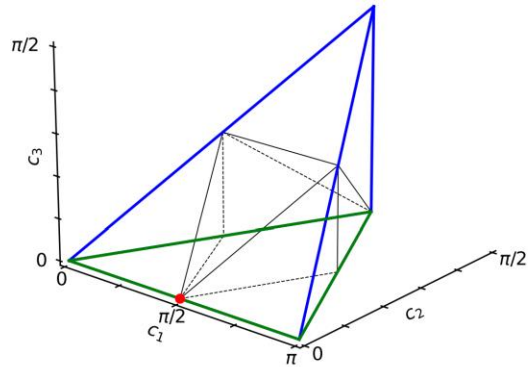
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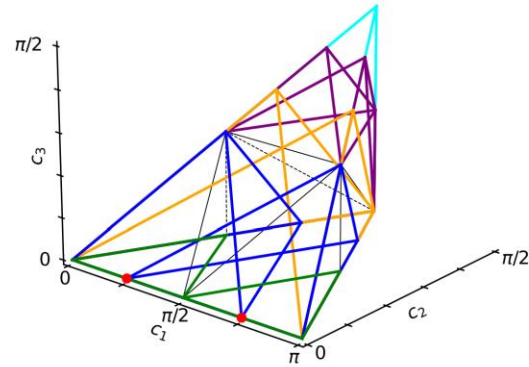
(a) iSWAP



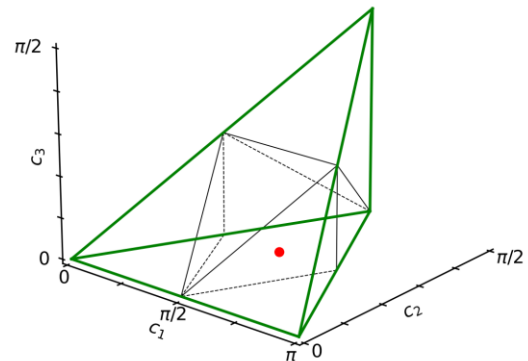
(b)  $\sqrt{i\text{SWAP}}$



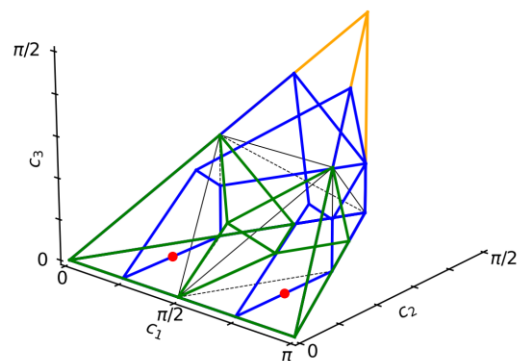
(c) CNOT



(d)  $\sqrt{\text{CNOT}}$



(e) B



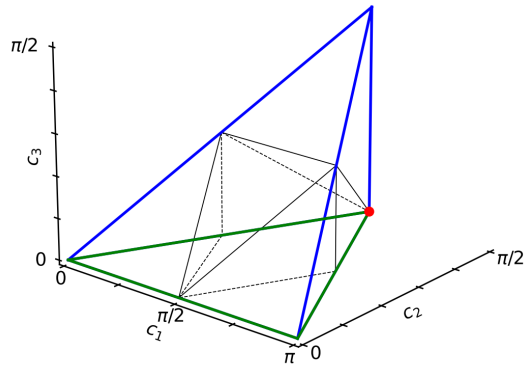
(f)  $\sqrt{B}$

- **Monodromy polytopes** finds minimum gate applications for any 2Q target gate
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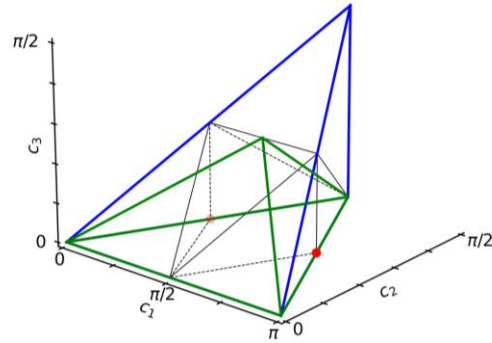
Decomposition *gate count* costs

Basis	iSWAP	$\sqrt{i\text{SWAP}}$	CNOT	$\sqrt{\text{CNOT}}$	B	$\sqrt{B}$
CNOT	2.0	2.0	1.0	2.0	2.0	2.0
SWAP	3.0	3.0	3.0	6.0	2.0	4.0
Haar	3.0	2.2	3.0	3.5	2.0	3.1

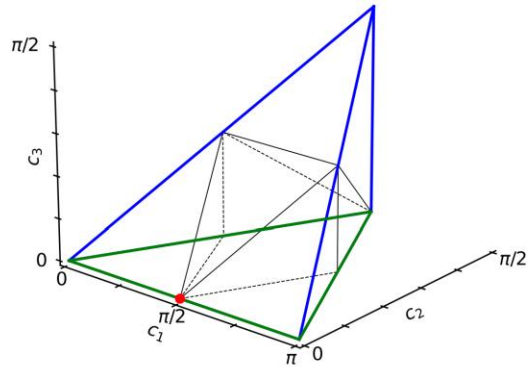
# Basis coverage volumes



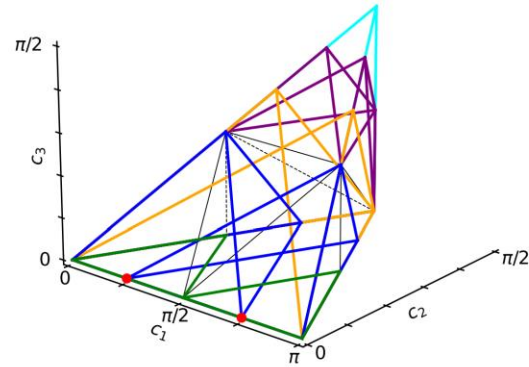
(a) iSWAP



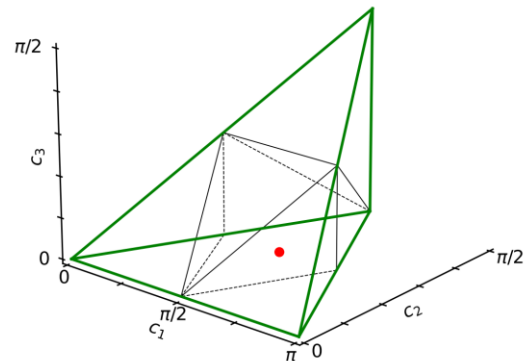
(b)  $\sqrt{i\text{SWAP}}$



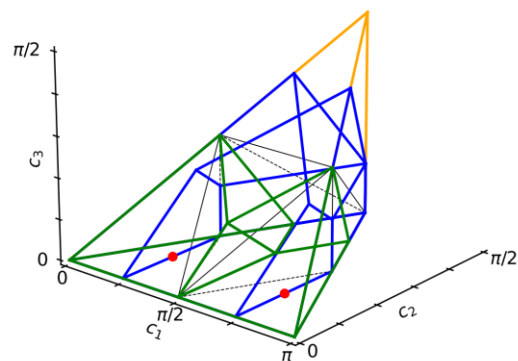
(c) CNOT



(d)  $\sqrt{\text{CNOT}}$



(e) B



(f)  $\sqrt{B}$

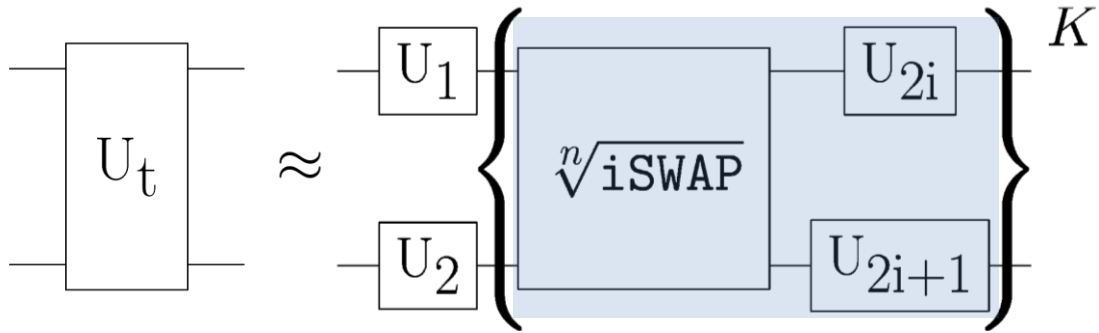
- **Monodromy polytopes** finds minimum gate applications for any 2Q target gate
- A single gate locally equivalent to itself
- SWAP is the most expensive target

Decomposition *gate count* costs

Basis	iSWAP	$\sqrt{i\text{SWAP}}$	CNOT	$\sqrt{\text{CNOT}}$	B	$\sqrt{B}$
CNOT	2.0	2.0	1.0	2.0	2.0	2.0
SWAP	3.0	3.0	3.0	6.0	2.0	4.0
Haar	3.0	2.2	3.0	3.5	2.0	3.1

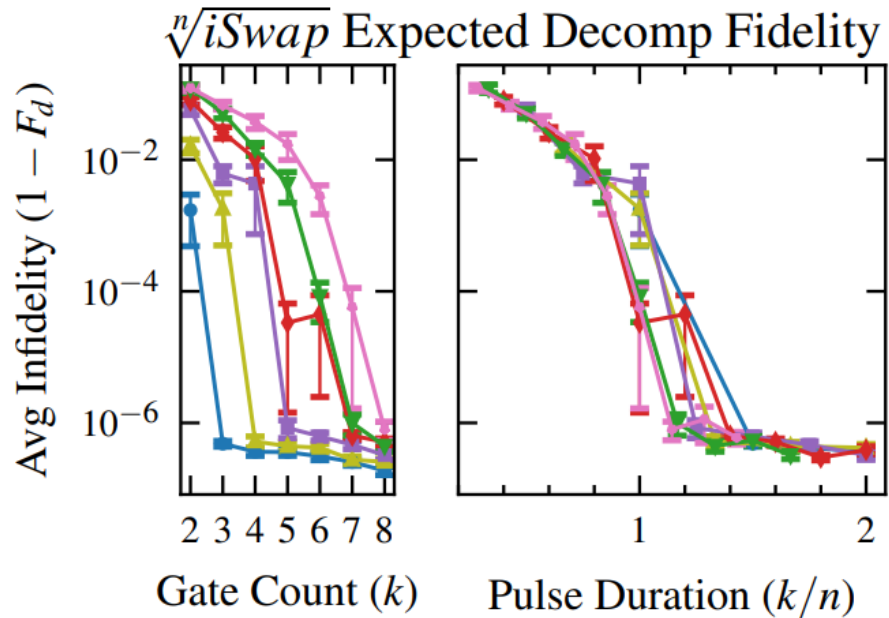
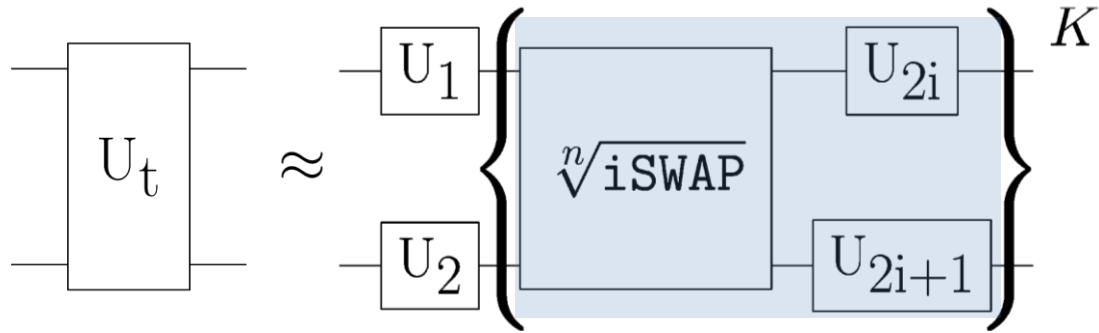
# Approximate decomposition into continuous iSWAP

Repeats  $K$  times |  $i \rightarrow 1$  to  $K$



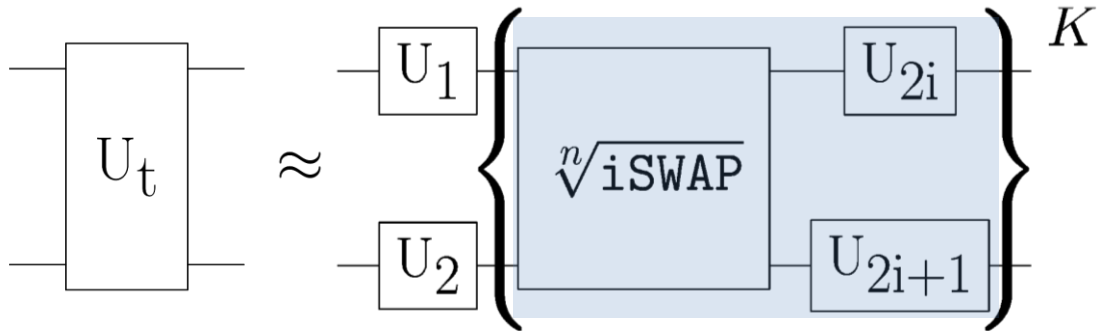
# Approximate decomposition into continuous iSWAP

Repeats  $K$  times |  $i \rightarrow 1$  to  $K$



# Approximate decomposition into continuous iSWAP

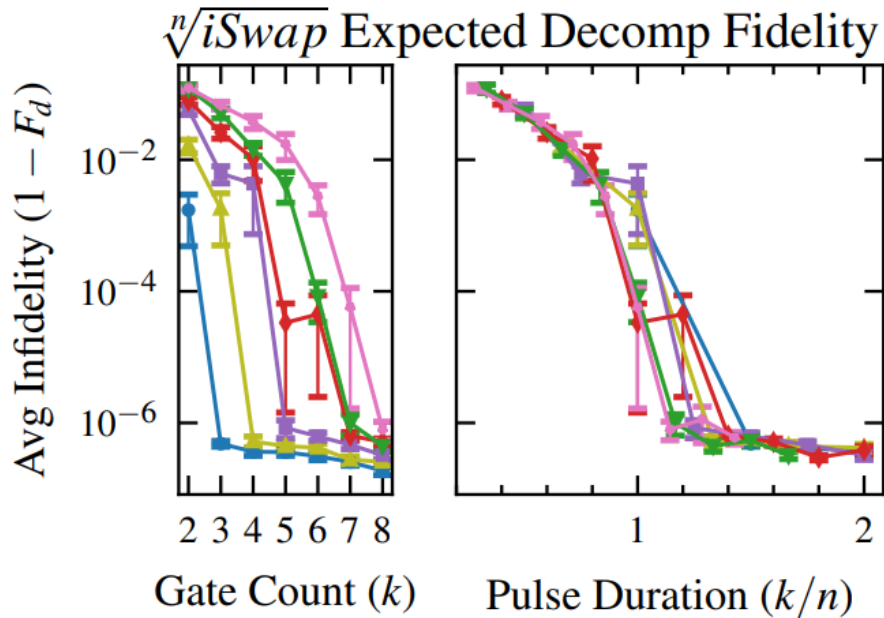
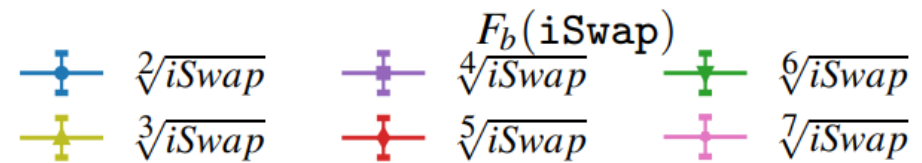
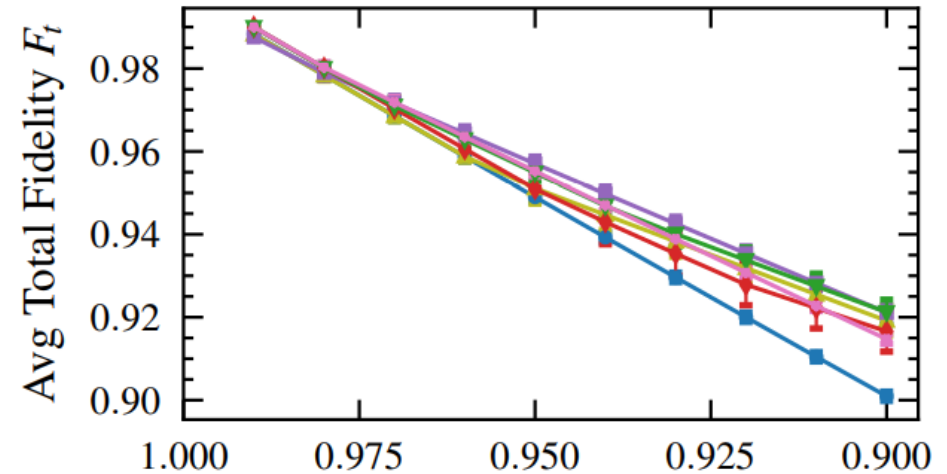
Repeats  $K$  times |  $i \rightarrow 1$  to  $K$



➤ Coherence-limited fidelity as a function of gate duration

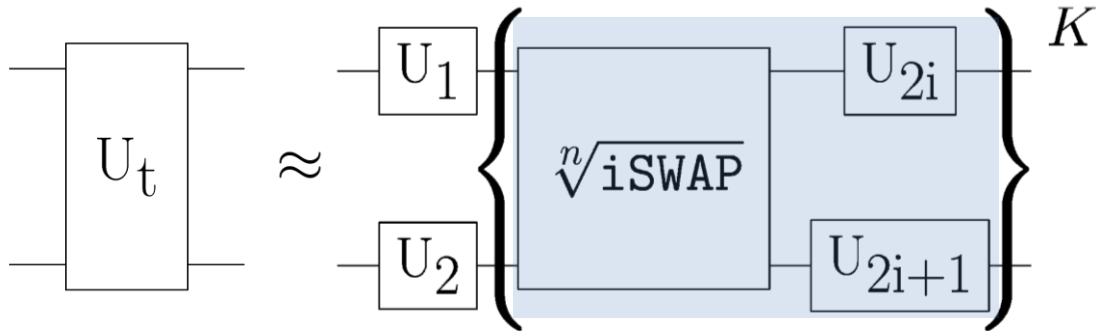
$$F(\sqrt[n]{i\text{SWAP}}) = 1 - \frac{1 - F(i\text{SWAP})}{n}$$

$\sqrt[n]{i\text{Swap}}$  Expected Total Fidelity



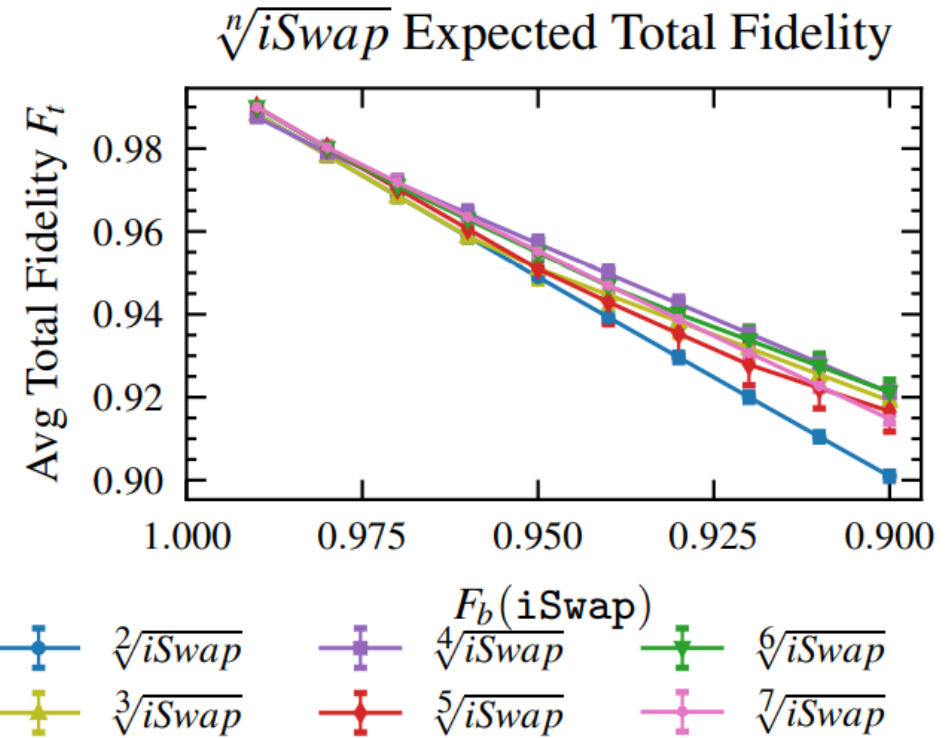
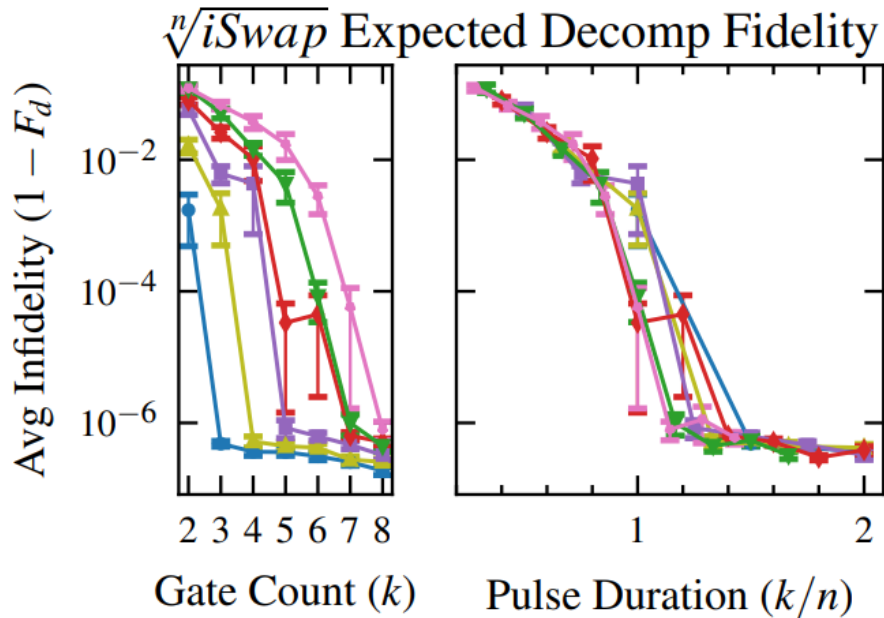
# Approximate decomposition into continuous iSWAP

Repeats  $K$  times |  $i \rightarrow 1$  to  $K$



➤ Coherence-limited fidelity as a function of gate duration

$$F(\sqrt[n]{i\text{SWAP}}) = 1 - \frac{1 - F(i\text{SWAP})}{n}$$



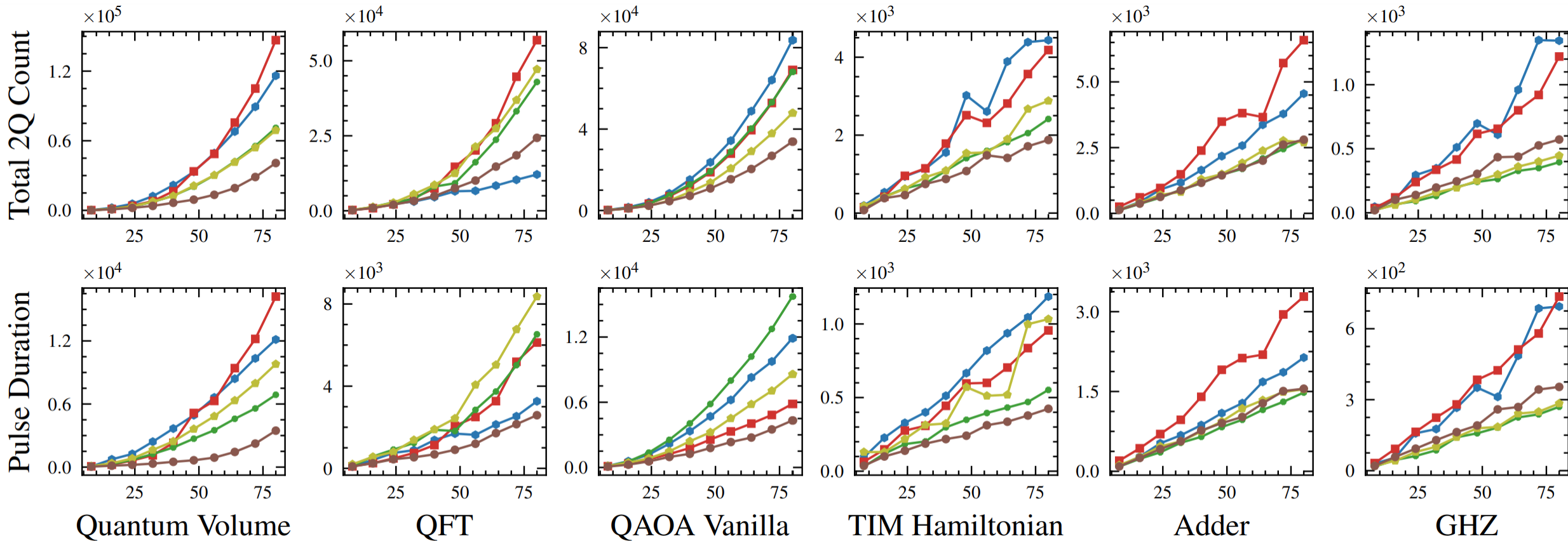
$\sqrt[2]{i\text{SWAP}}$  decreases infidelity by 51%,  $\sqrt[4]{i\text{SWAP}}$  by 58% vs iSWAP



◆ Heavy-Hex-CX  
■ Square-Lattice-SYC

# Benchmarking duration

◆ Tree- $\sqrt{iSWAP}$   
◆ Tree-I- $\sqrt{iSWAP}$   
● Hypercube- $\sqrt{iSWAP}$



- Considering the impact from both topology and basis gate**  
 80Q Heavy-Hex 54% slower ( $\ll$  fidelity) vs. Hypercubes

Connectivity Topologies

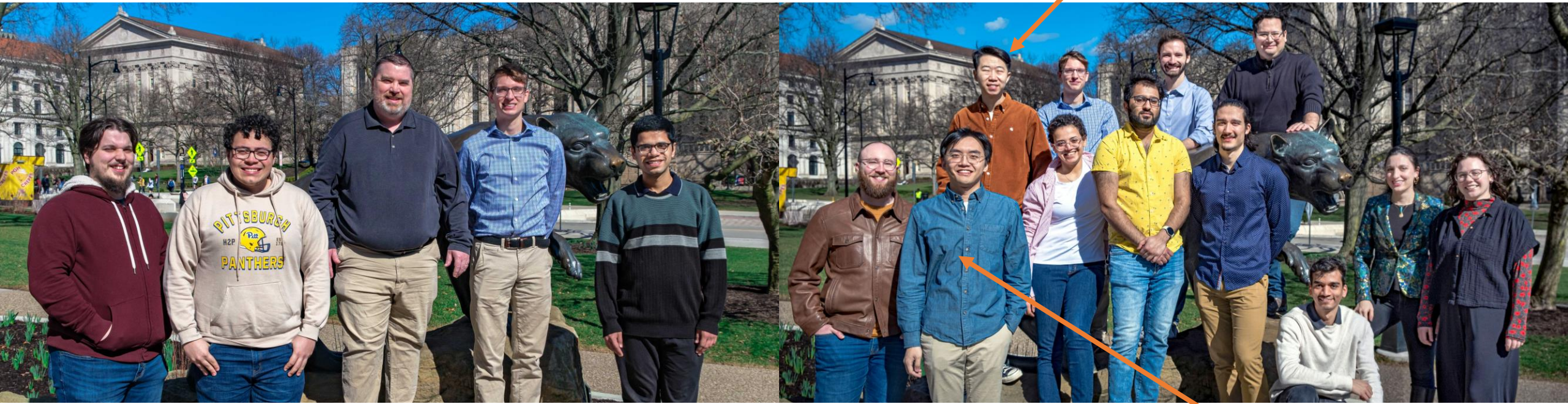


Native Hardware Gates

# Conclusion

1. SNAIL coupling provides both powerful topologies and basis gates
2. Corral increases parallelism for near-term quantum applications
3. Continuous iSwap gate set shortens overall duration.

McKinney, et al. **HPCA** (2023).



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