Co-designed architectures for modular superconducting quantum computers

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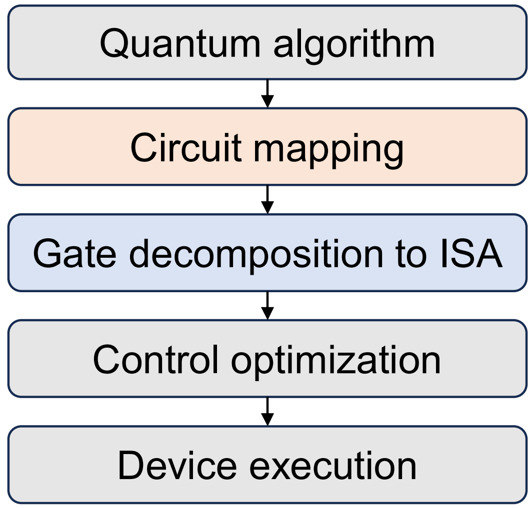
YQI, September 2024

Transpilation as co-design framework



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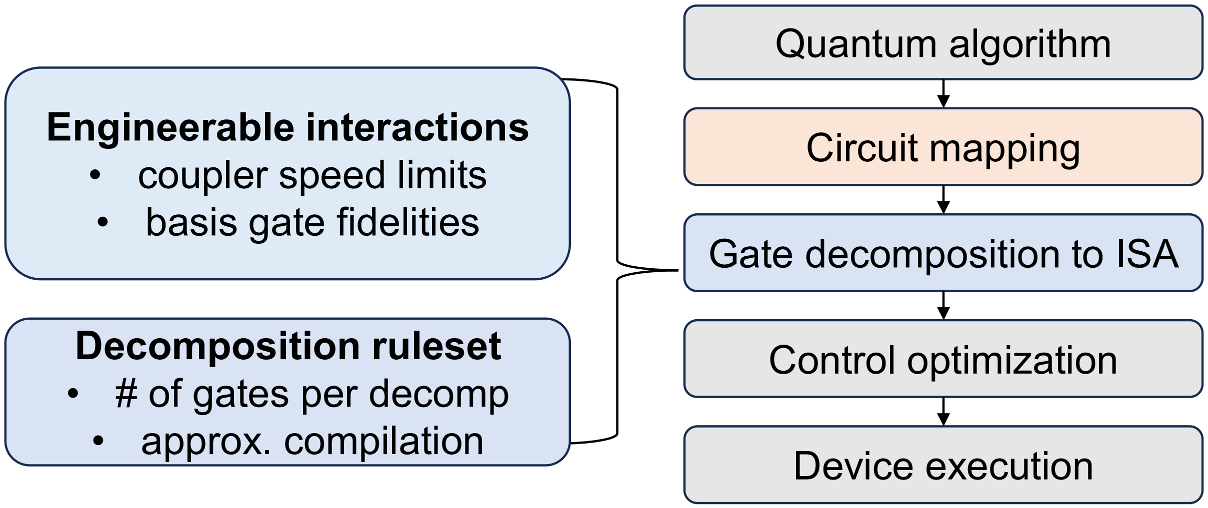
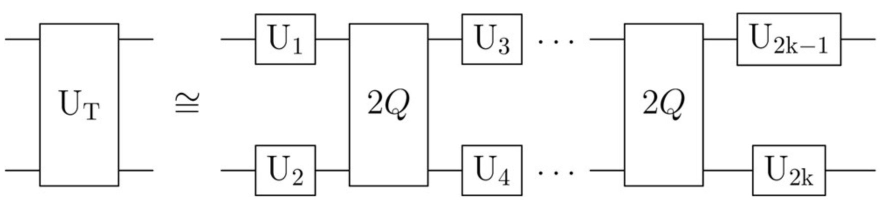




Transpilation as co-design framework

Engineerable interactions
- coupler speed limits
- basis gate fidelities

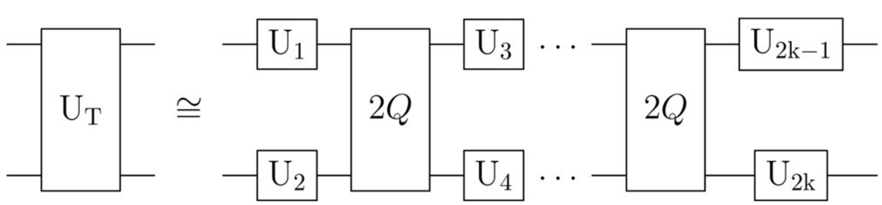
Decomposition ruleset
- # of gates per decomp
- approx. compilation

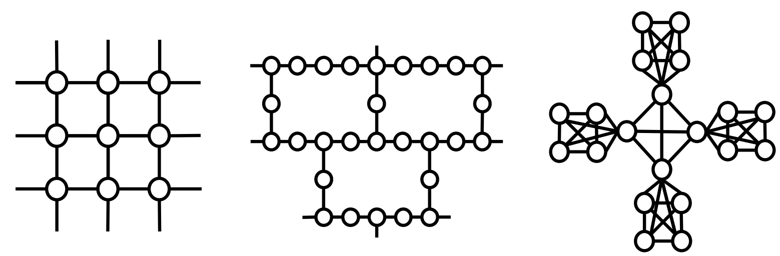
Quantum algorithm
↓
Circuit mapping
↓
Gate decomposition to ISA
↓
Control optimization
↓
Device execution

Device Constraints
- mode frequency bandwidth
- fabrication precision
- hybridization strengths
- coupler speed-limit

Gate fidelity threshold
- # qubits per module
- qubit-coupler connectivity

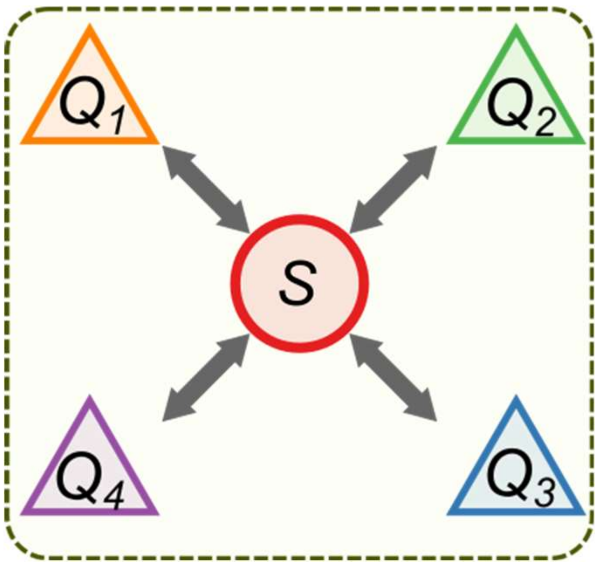
Qubit frequency allocation
- # unique qubit frequencies

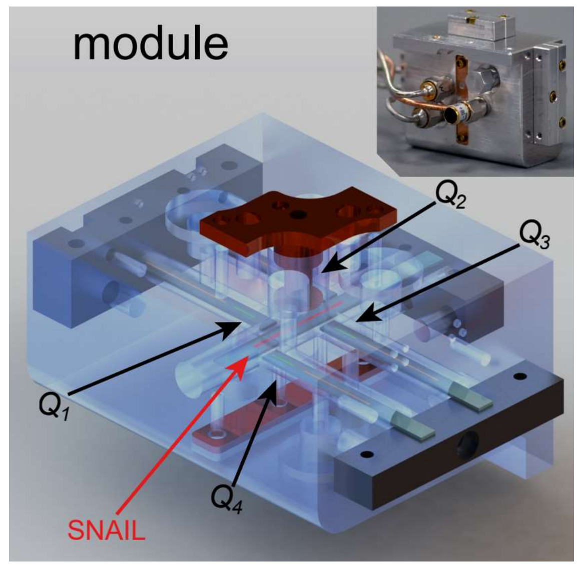


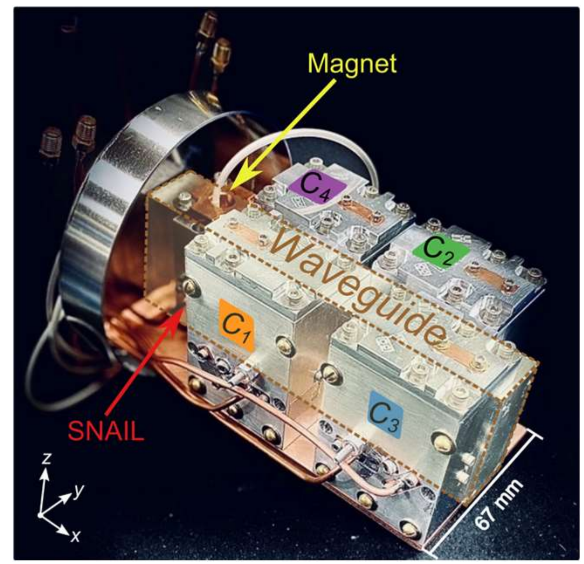


SNAIL module

Four qubit SNAIL-based quantum module

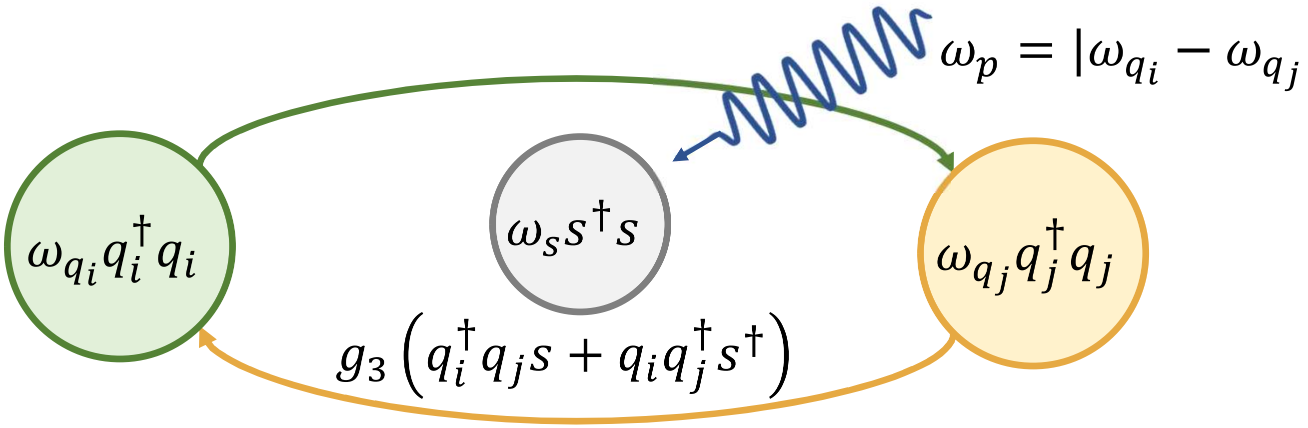






Xia, et al. **arXiv:2306.10162** (2023)
Zhou, et al. **npj Quantum Inf 9**, 54 (2023)

Parametric two qubit gates

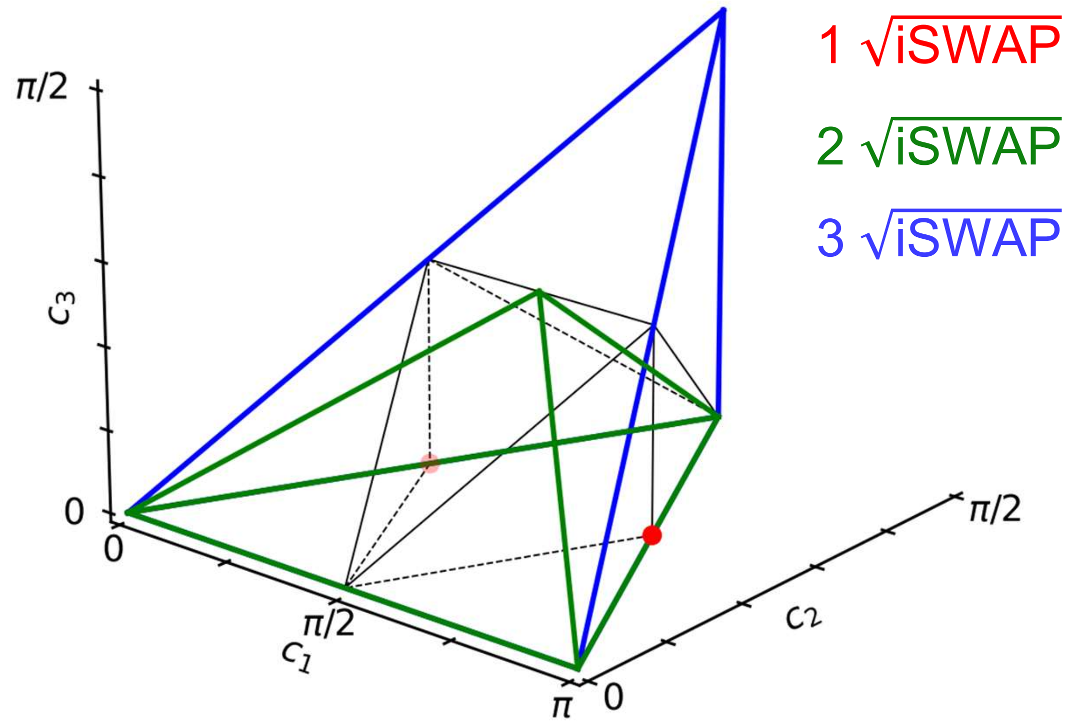


$$\frac{H_{eff}}{\hbar} = g_2^{eff}\left(a^\dagger b + a b^\dagger\right), \qquad where \quad g_2^{eff} = g_3\left(\frac{g}{\Delta}\right)^2 \sqrt{n_p}$$

Clerk, et al. **Reviews of Modern Physics** (2010)
Bergeal, et al. **Nature Physics** (2010)
Frattini, et al. **Applied Physics Letters** (2017)

Efficient instruction sets using $\sqrt{\text{iSWAP}}$

$$\sqrt{\texttt{iSWAP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



McKinney, et al. **ISCA** (2023)
Huang, et al. **Physical Review Letters** (2023)
Chen, et al. **arxiv:2312.05652** (2023)

Geometrically representing quantum gates

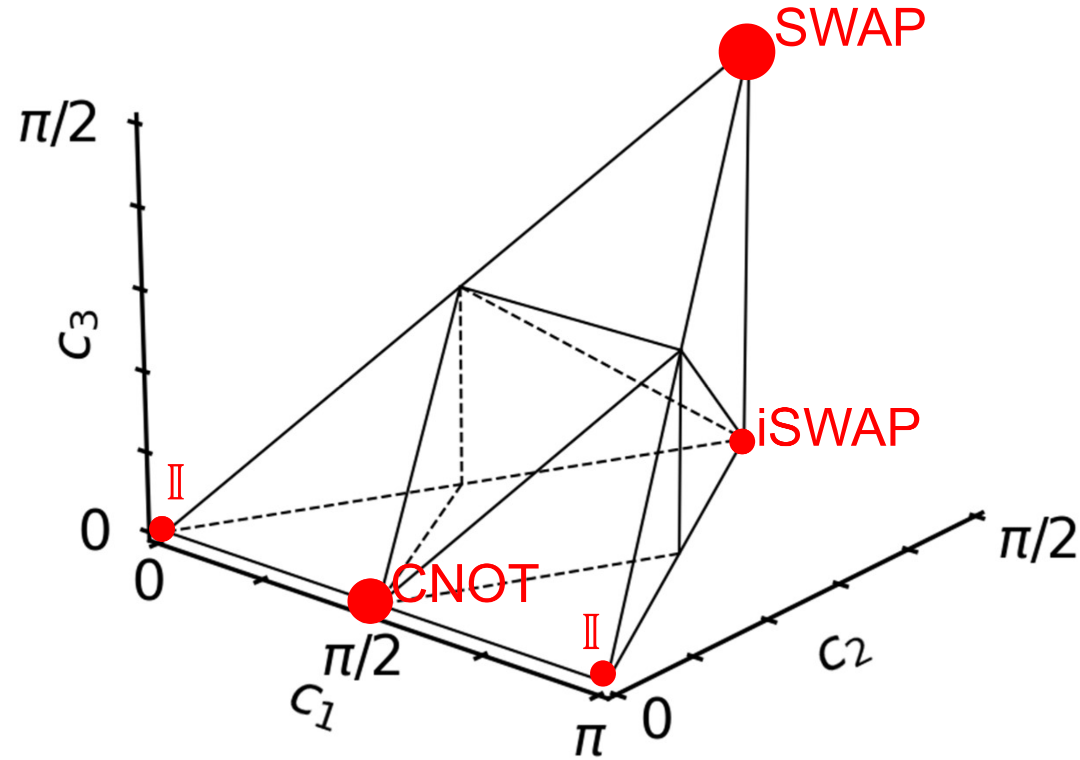
Quantum control

↓

Unitary gate

↓

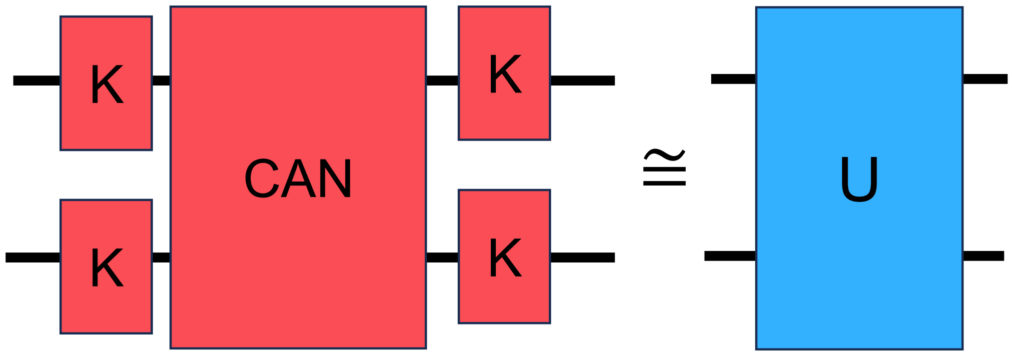
Weyl chamber coordinate



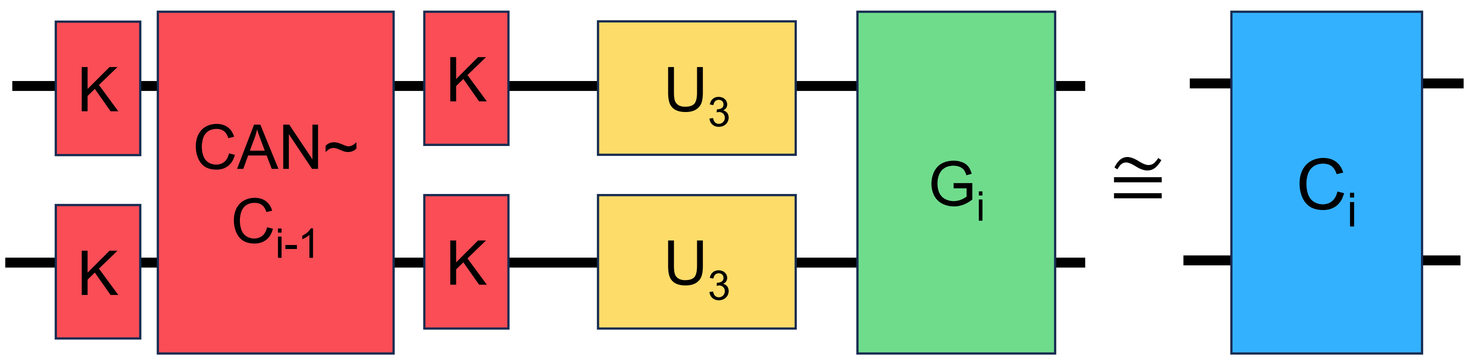
Peterson, et al. **Quantum 4** (2020)
Makhlin, Yuriy. **Quantum Information Processing** (2002)
Watts, et al. **Entropy 15** (2013)
Zhang, et al. **Physical Review A** (2003)

Cartan KAK decomposition

All 2Q gates can be specified by 3 invariants up to local rotations (1Q)



$$U = (K_1 \otimes K_2) e^{-i(k_x \sigma_{XX} + k_y \sigma_{YY} + k_z \sigma_{ZZ})} (K_3 \otimes K_4)$$



Conversion/Gain candidate basis gates

- **Engineerable interactions from 3-wave mixing**

$$\hat{H} = g_c(e^{i\phi_c}a^\dagger b + e^{-i\phi_c}ab^\dagger) + g_g(e^{i\phi_g}ab + e^{-i\phi_g}a^\dagger b^\dagger)$$

- Native gates limited to XX,YY components

$$\hat{H} = \frac{1}{2}[(g_c + g_g)XX + (g_c - g_g)YY]$$

- Cartan trajectories from Makhlin invariants

$$G_1 = (\cos(2g_c t) + \cos(2g_g t))^2/4$$

$$G_2 = \cos(2(g_c - g_g)t) + \cos(2(g_c + g_g)t) + 1$$

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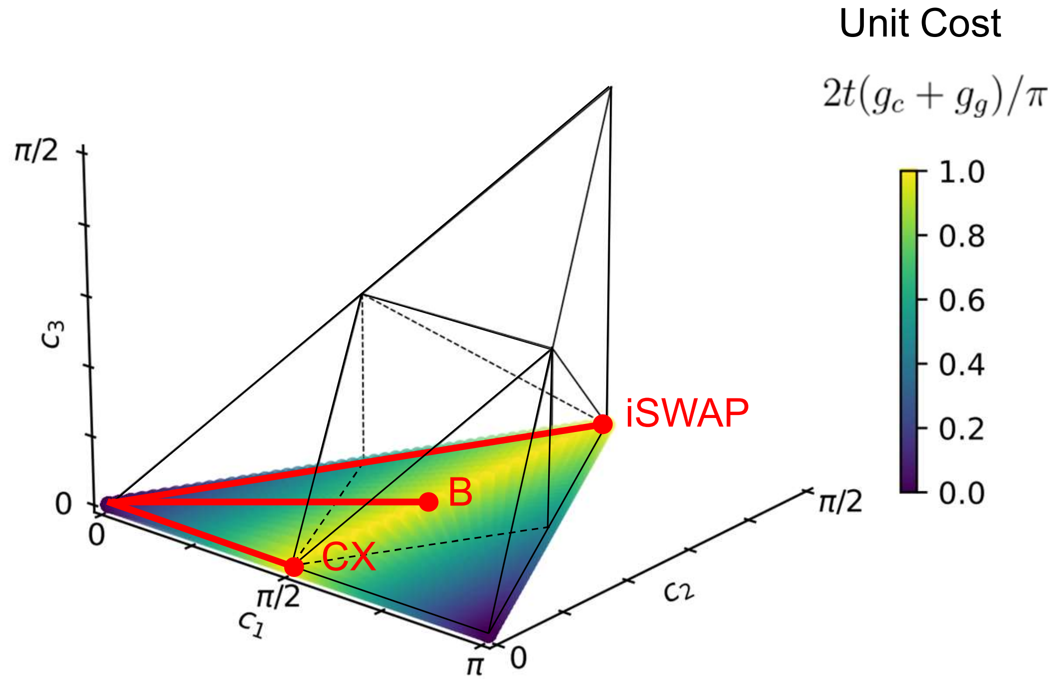


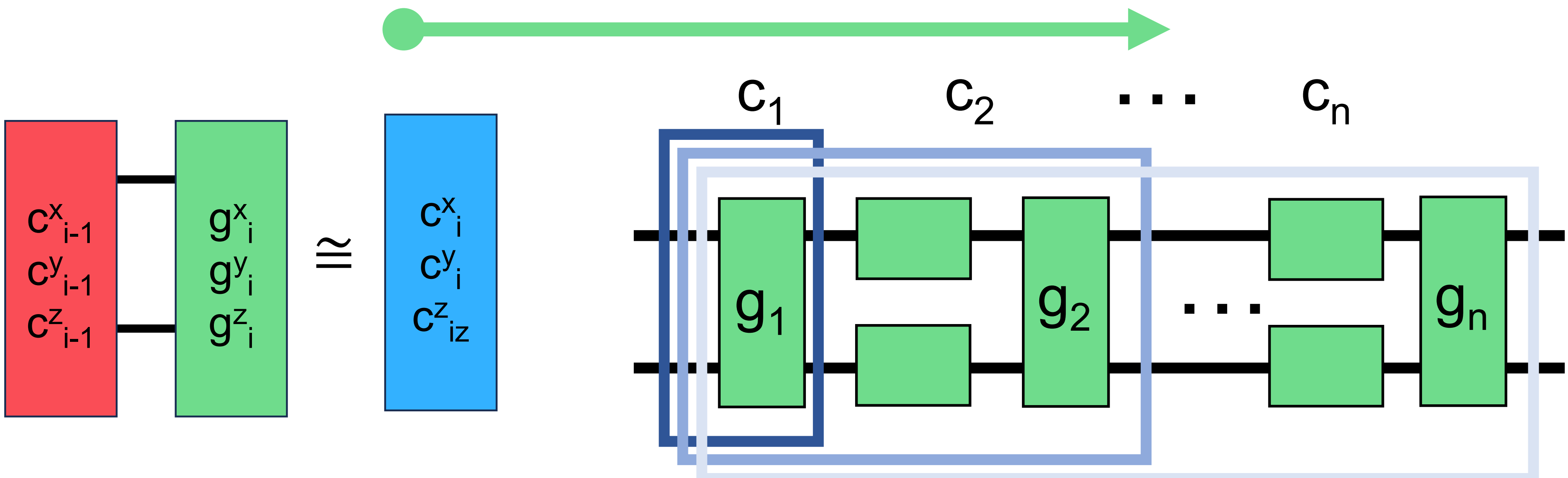
Monodromy polytopes

Satisfying all 72 **quantum Littlewood-Richardson** linear inequalities implies

given a circuit with invariant $\mathbf{c}_{i-1}$,

appending $\mathbf{g}_i$, a gate from the device's instruction set,

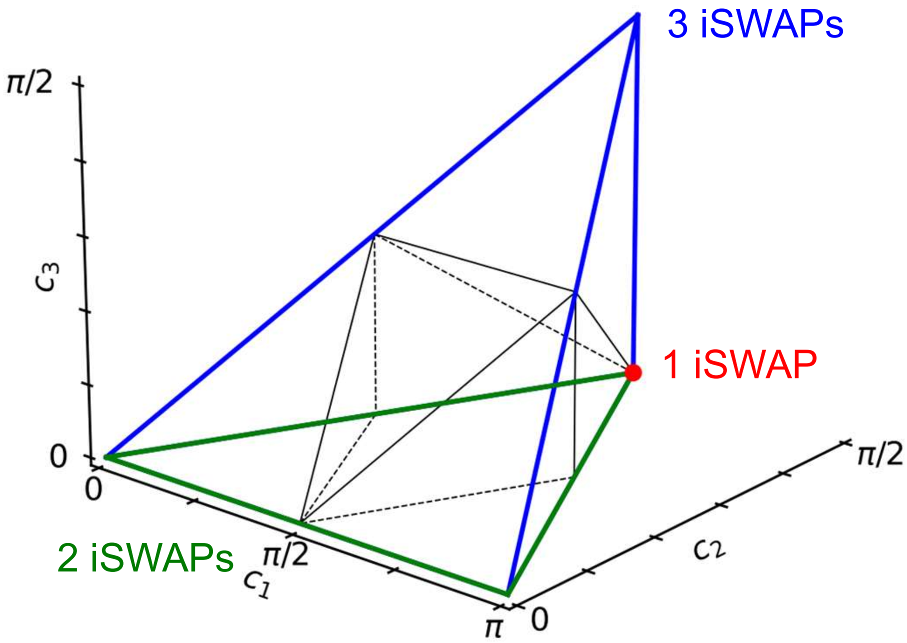
can yield a circuit with invariant $\mathbf{c}_i$.



Satisfy all $\mathbf{L}_i(\mathbf{c}_{i-1}, \mathbf{g}_i, \mathbf{c}_i)$ such that $\forall i, g_i \in \mathrm{ISA}$

Peterson, et al. ***Quantum* 4** (2020)

Basis coverage volumes



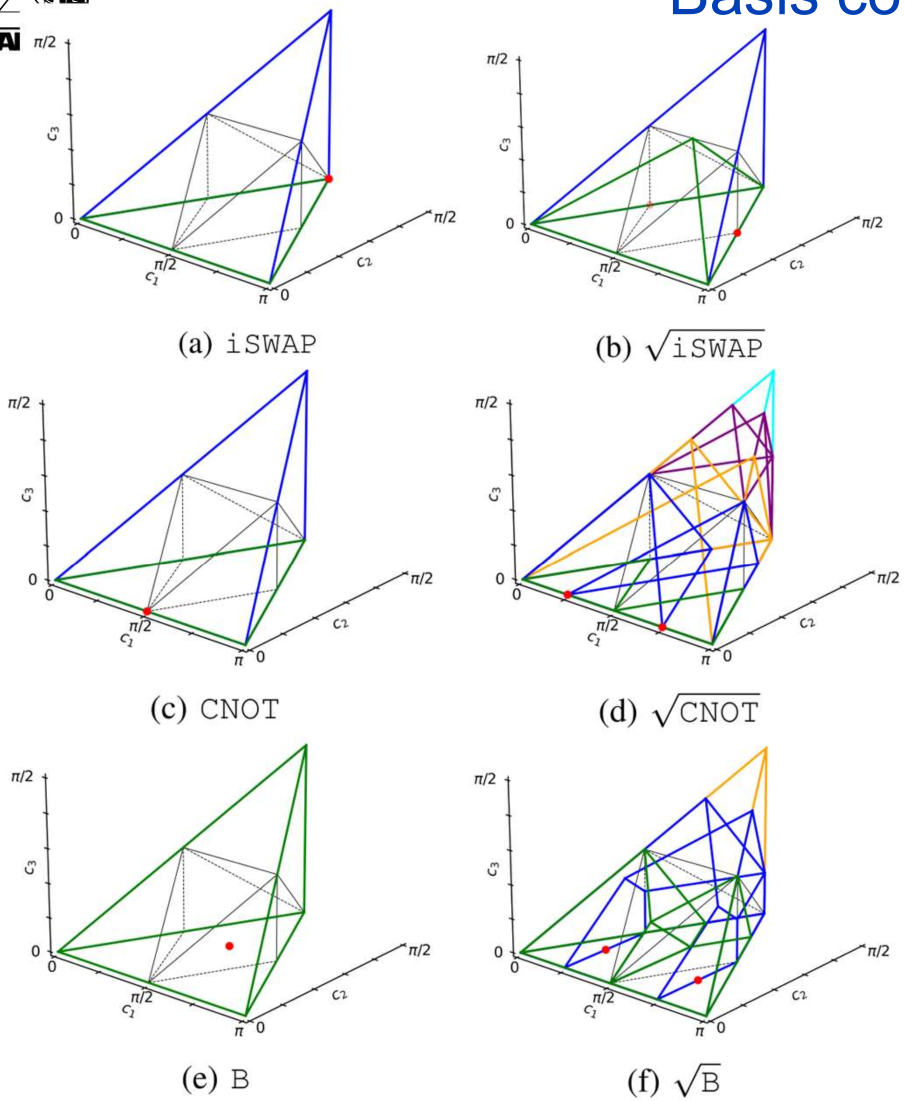
- **Monodromy polytopes** finds minimum gate applications for any 2Q target gate!

Target\Basis	iSWAP
CNOT	2.0
SWAP	3.0
Haar	3.0

Peterson, et al. ***Quantum* 4** (2020)

McKinney, et al. **ISCA** (2023)

Basis coverage volumes



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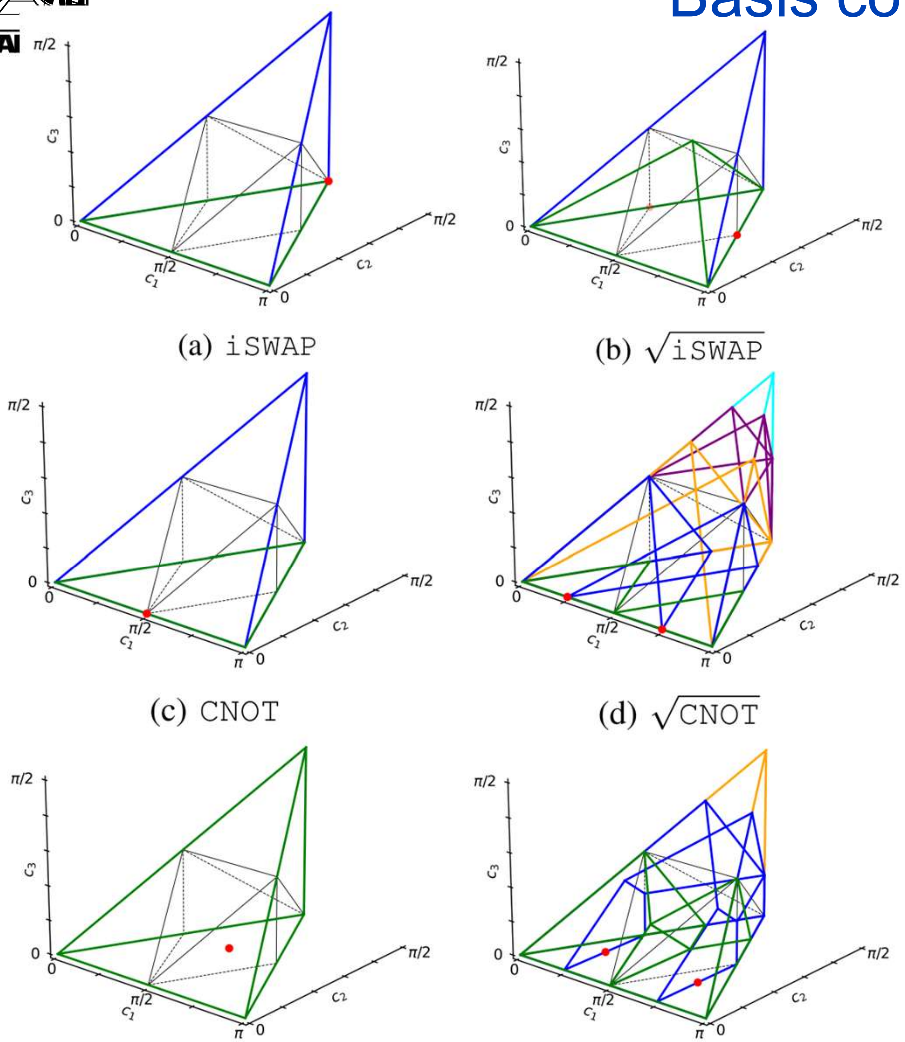
Decomposition *gate count* costs

Target\Basis	iSWAP	$\sqrt{\text{iSWAP}}$	CX	$\sqrt{\text{CX}}$	B	$\sqrt{\text{B}}$
CNOT	2.0	2.0	1.0	2.0	2.0	2.0
SWAP	3.0	3.0	3.0	6.0	2.0	4.0
Haar	3.0	2.2	3.0	3.5	2.0	3.1

Peterson, et al. ***Quantum* 4** (2020)

McKinney, et al. **ISCA** (2023)

Basis coverage volumes



(a) iSWAP (b) $\sqrt{\text{iSWAP}}$ (c) CNOT (d) $\sqrt{\text{CNOT}}$ (e) B (f) $\sqrt{\text{B}}$

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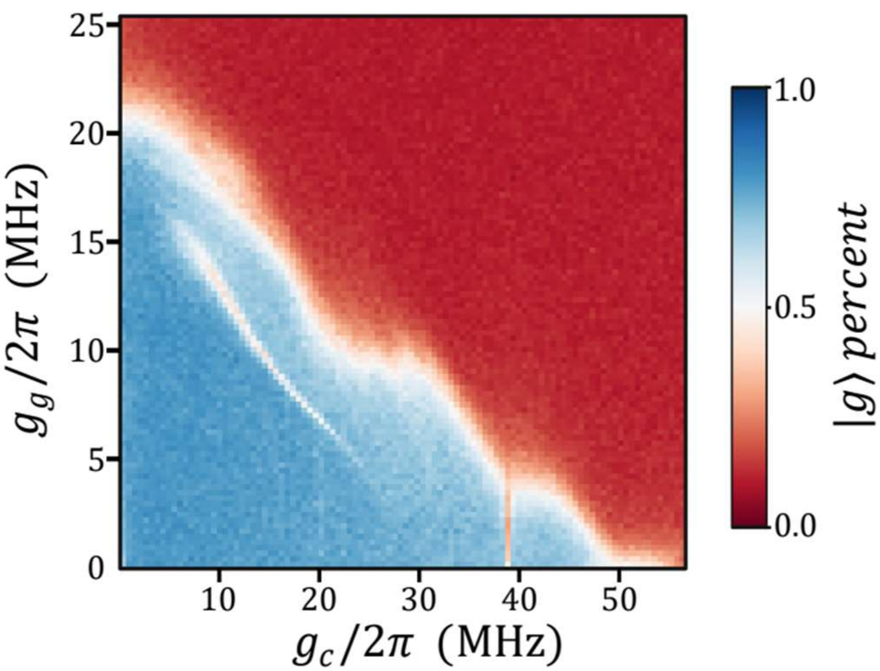
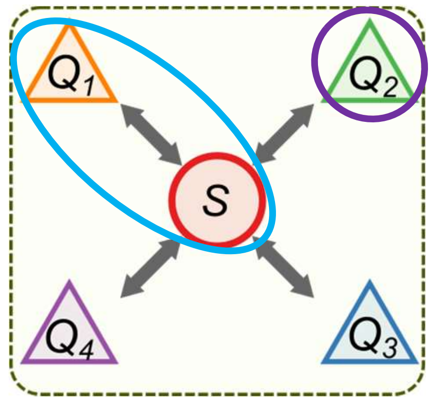
Hardware speed limits



Limitation of SNAIL when driving both gain and conversion

Module

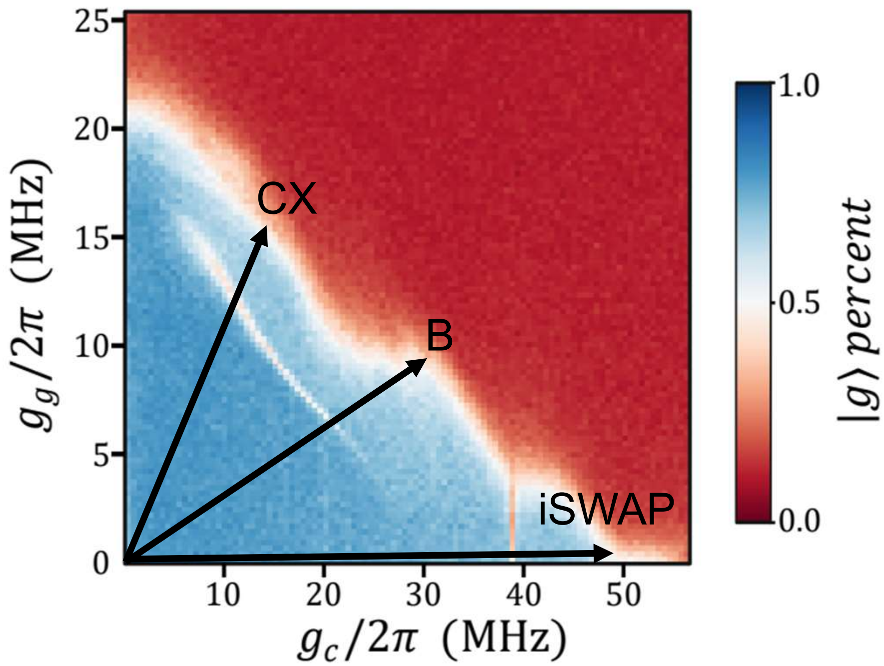


Drives applied between SNAIL and qubit

Measure second qubit to witness SNAIL breakpoint

Zhou, et al. **npj Quantum Inf 9**, 54 (2023).

Hardware speed limits



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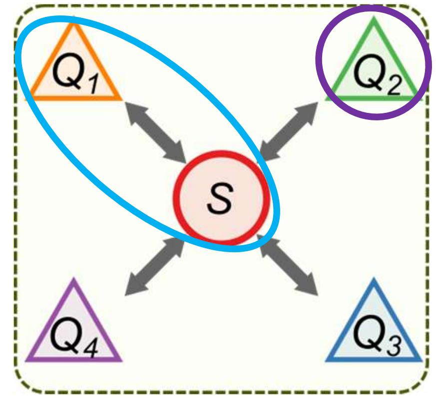

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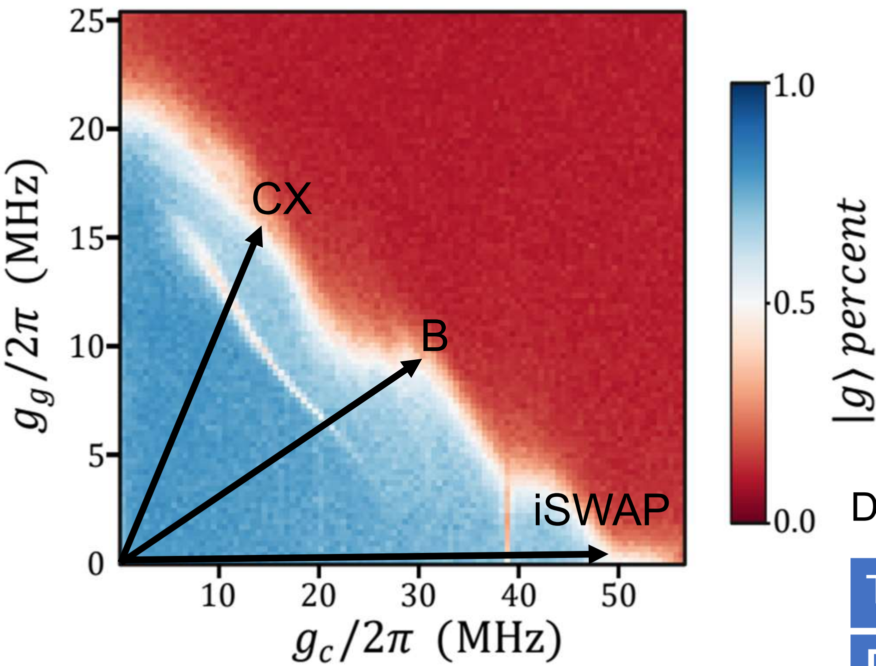

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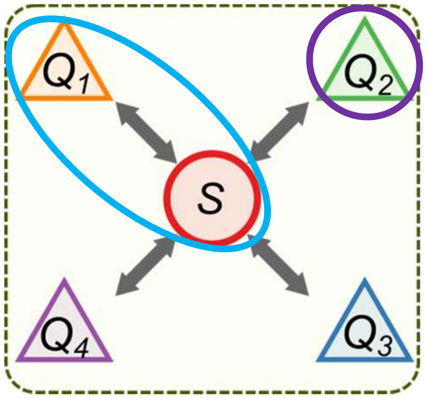

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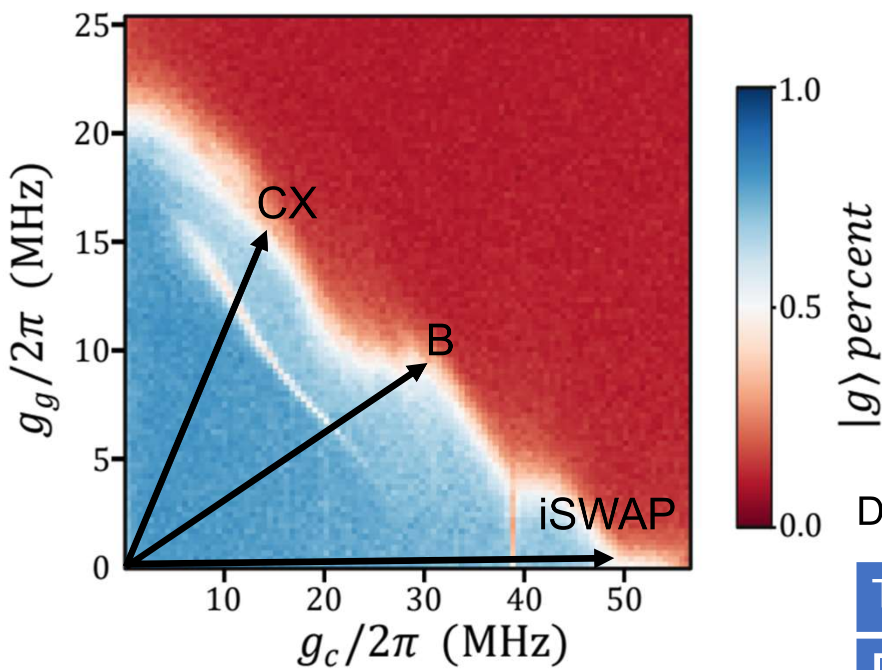
Duration	**1.0**	**0.5**	**1.8**	**0.9**	**1.4**	**0.7**
CNOT	2.0	1.0	1.8	1.8	2.8	1.4
SWAP	3.0	1.5	5.4	5.4	2.8	2.8
Haar	3.0	1.1	5.4	3.2	2.8	2.2

Hardware speed limits



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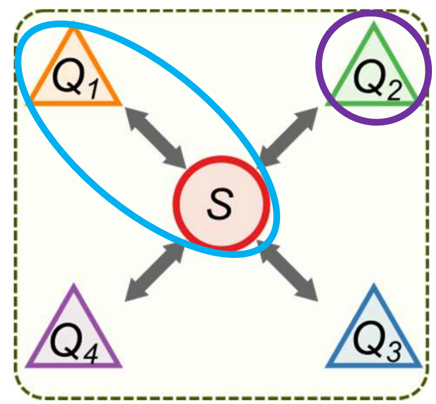

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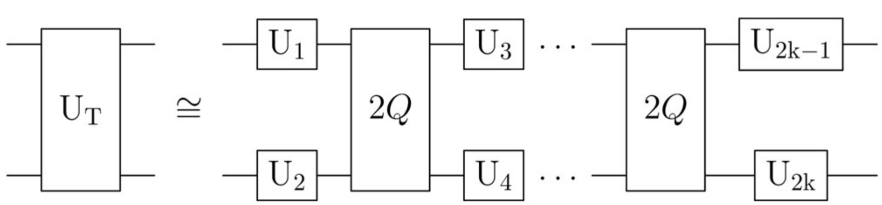
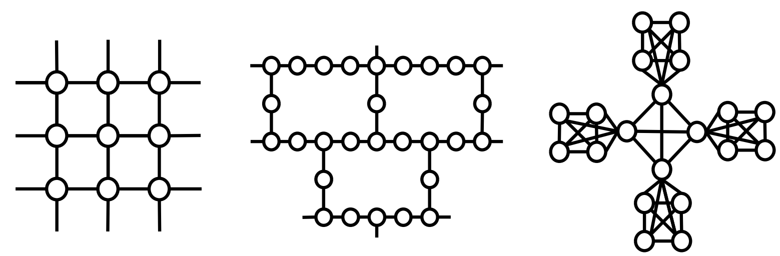
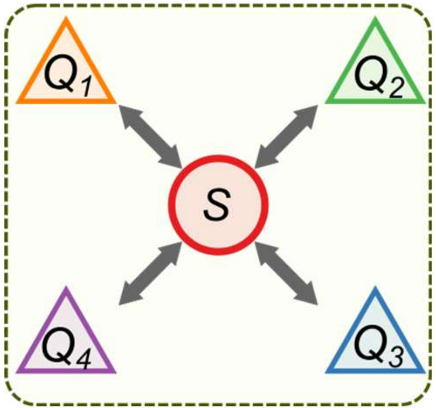




Extending the module



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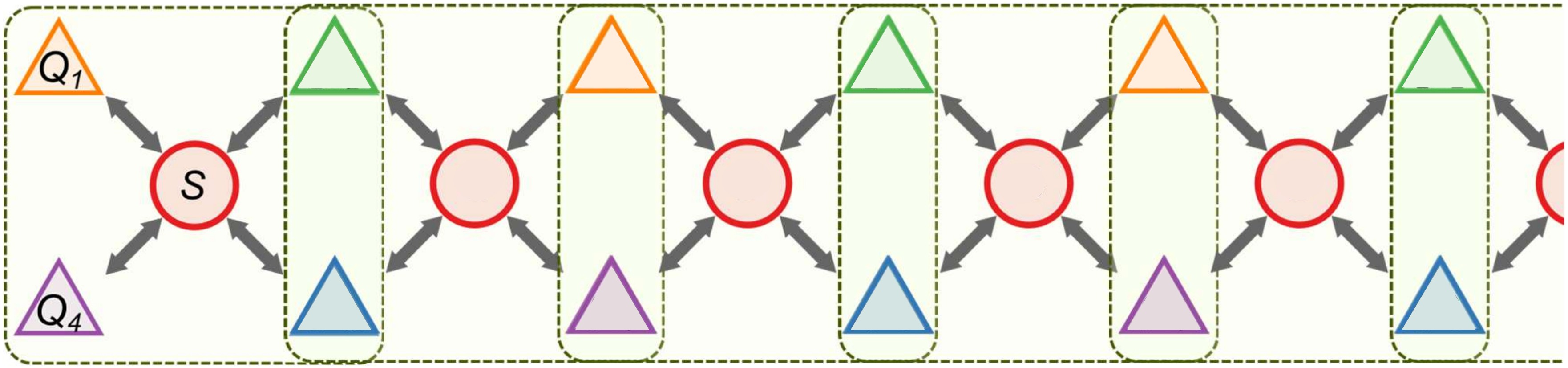
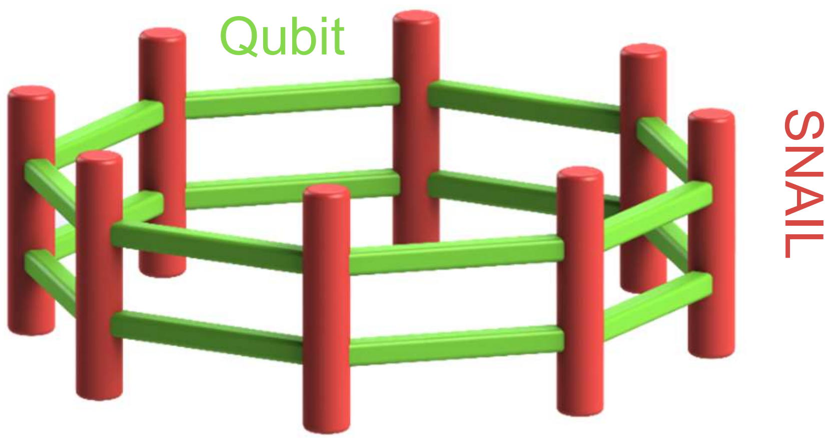
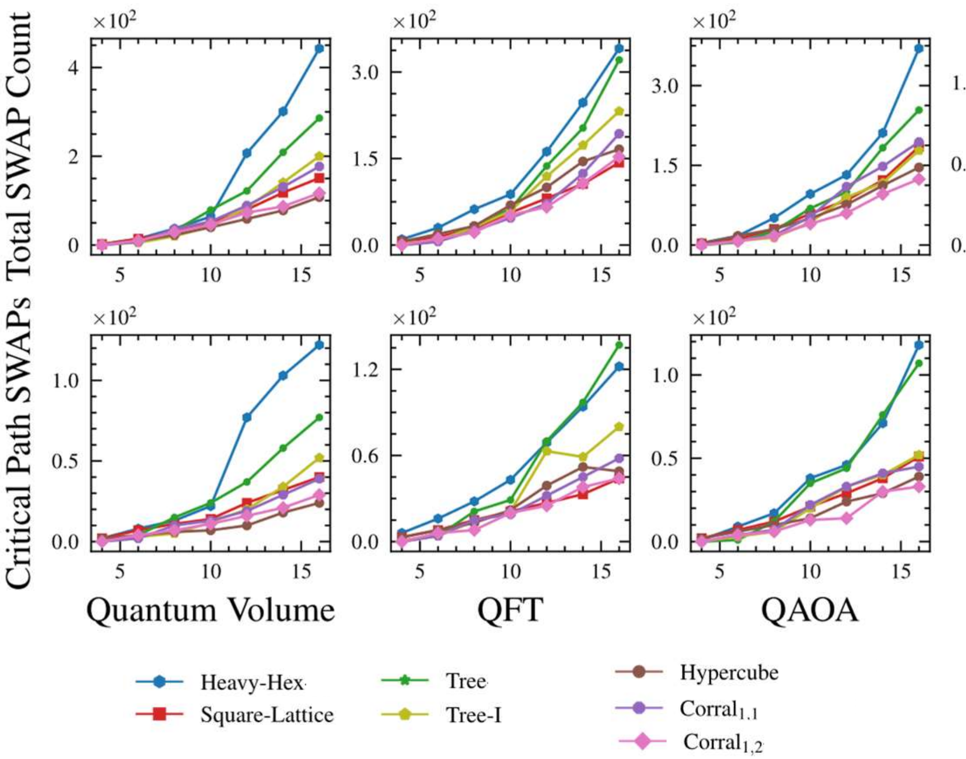
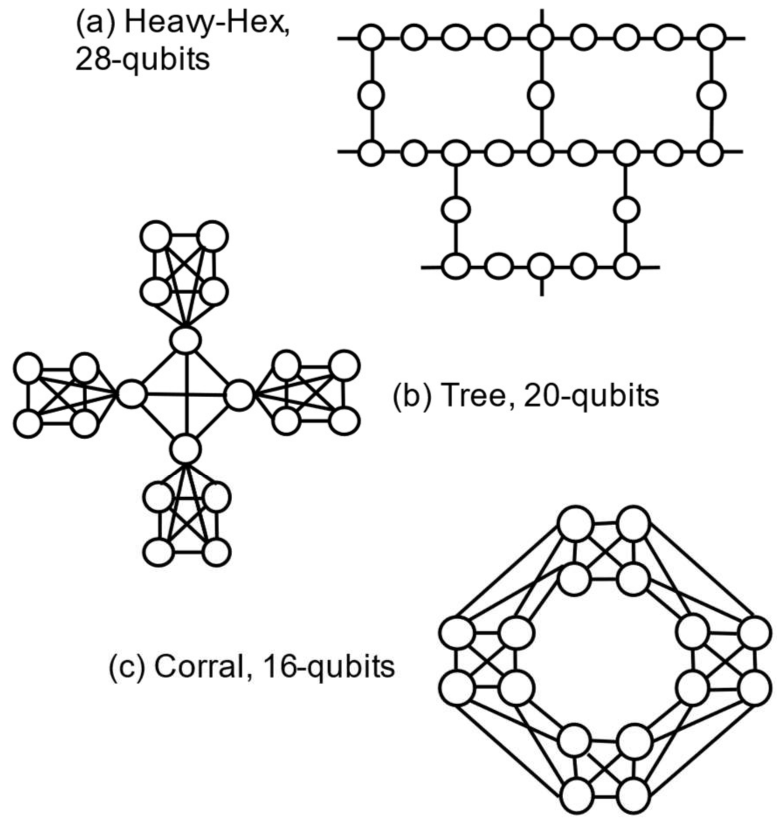




Connectivity reduces circuit costs

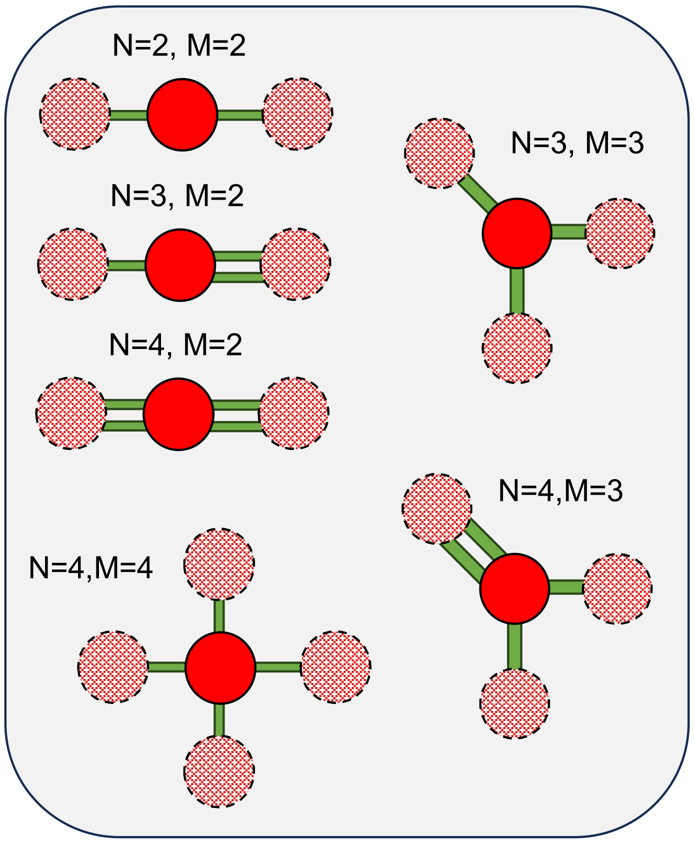




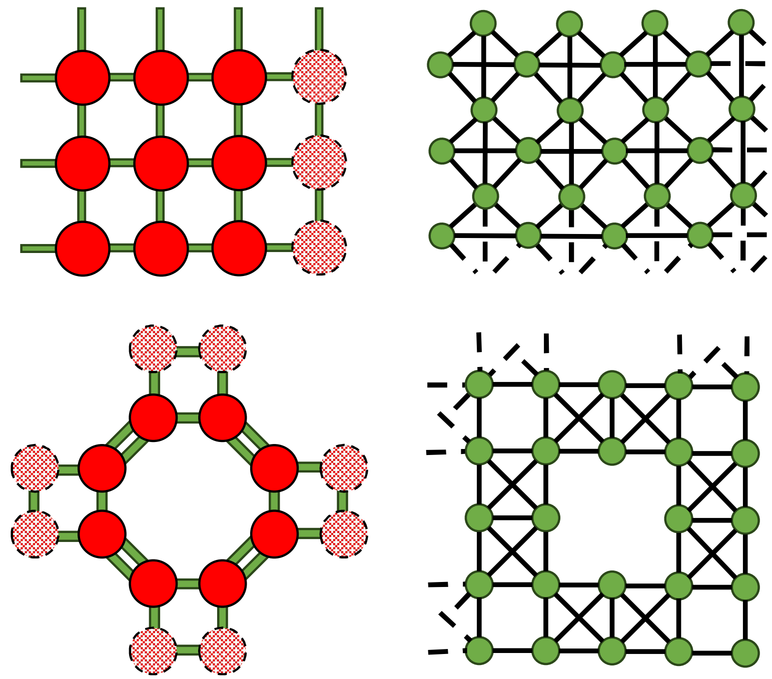
McKinney, et al. **HPCA** (2023)

Alternative designs?

SNAIL and qubit frequencies **must yield unique sets of parametric drives** per neighborhoods

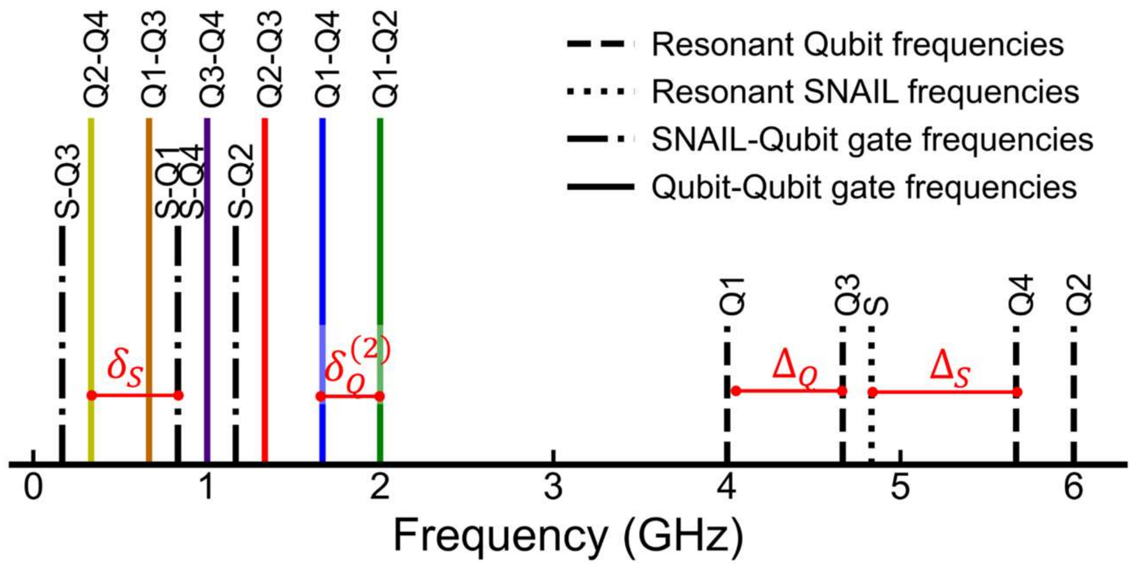


Maximum incident edges per node (N)?
Maximum adjacent vertices (M)?
Maximum vertex variants (k)?



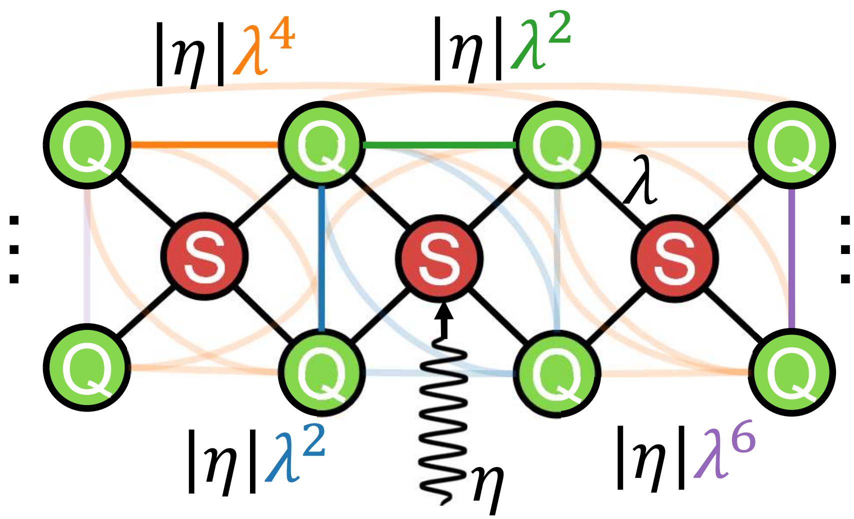
Spectral crowding of terms

$$\tilde{H}_I = g_3 \left(s e^{-i\tilde{\omega}_s t} + \eta e^{-i\omega_p t} + \sum_i \lambda_{si} q_i e^{-i\tilde{\omega}_{q_i} t} + \text{h.c.} \right)^3$$
$$+ \sum_i \frac{\alpha_i}{12} \left(q_i e^{-i\tilde{\omega}_{q_i} t} - \lambda_{si} (s e^{-i\tilde{\omega}_s t} + \eta e^{-i\omega_p t}) + \text{h.c.} \right)^4$$



Spectator error budgeting

- Intra-module terms protected by frequency separation
- Inter-module terms protected by weak SNAIL-SNAIL hybridization



$$\eta = (\epsilon^x + i\epsilon^y)\frac{\omega_s}{(\omega_p^2 - \omega_s^2)}$$

$$H_{conv} = \sum_{i>j} q_i^\dagger q_j \left(6\eta g_3 \lambda^2 e^{-it(\omega_{q_i} - \omega_{q_j} + \omega_p)}\right) + \text{h.c.}$$

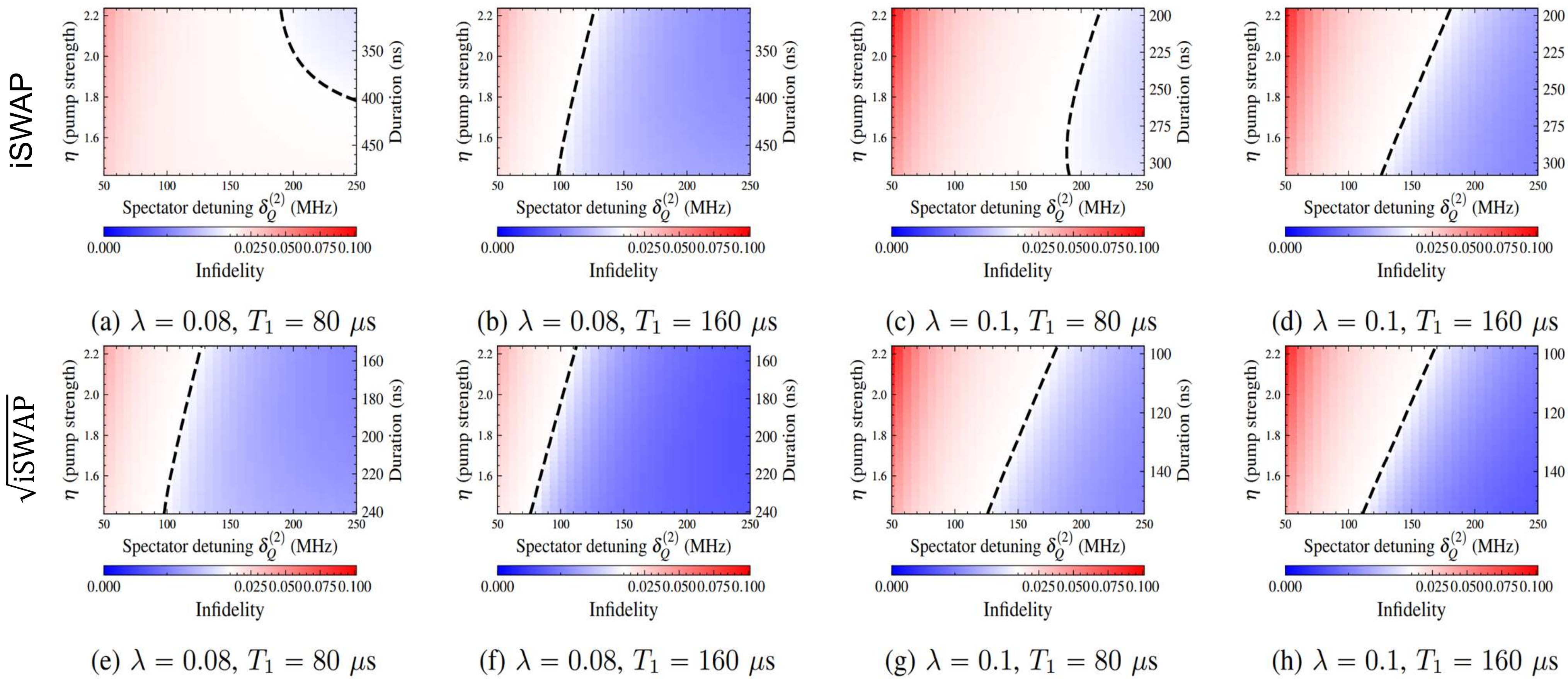
$$H(t) = 6|\eta| g_3 \lambda^2 \left(\mathbf{q_1^\dagger q_2} + \frac{2}{\delta_Q^{(2)} \log 2}\mathbf{q_3^\dagger q_4}\right)$$

When is RWA a good approximation?

Error budget as a separation design constraint

$$F_{\mathrm{avg}}(U,V) = \frac{16\,\mathrm{Tr}[UV^\dagger] + 1}{17}$$

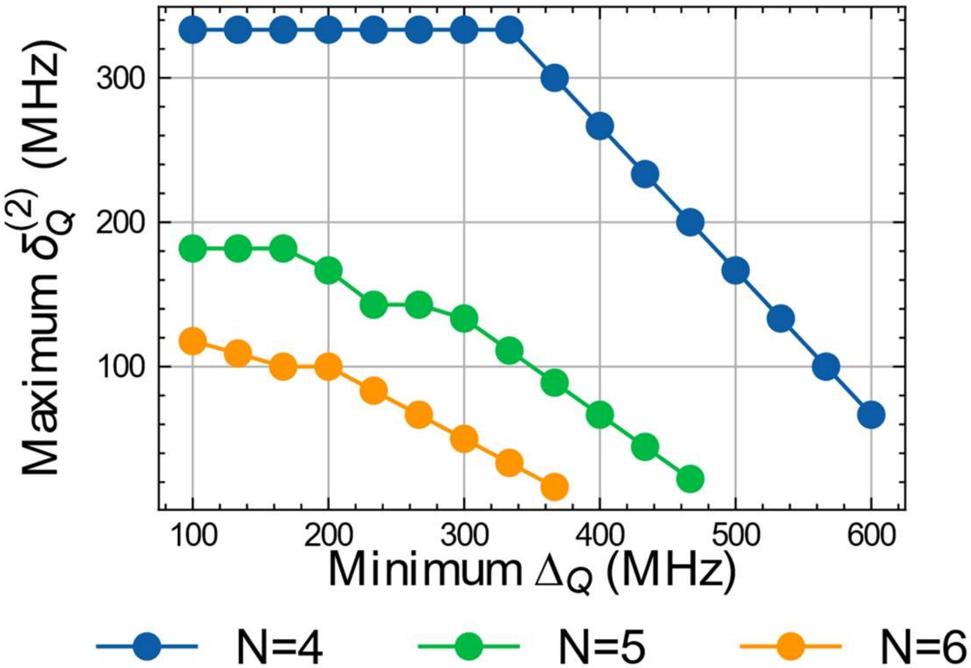
Threshold at F > .99



(a) $\lambda = 0.08,\ T_1 = 80\ \mu s$ (b) $\lambda = 0.08,\ T_1 = 160\ \mu s$ (c) $\lambda = 0.1,\ T_1 = 80\ \mu s$ (d) $\lambda = 0.1,\ T_1 = 160\ \mu s$

(e) $\lambda = 0.08,\ T_1 = 80\ \mu s$ (f) $\lambda = 0.08,\ T_1 = 160\ \mu s$ (g) $\lambda = 0.1,\ T_1 = 80\ \mu s$ (h) $\lambda = 0.1,\ T_1 = 160\ \mu s$

Satisfying constraints using linear programming



➢ Binary search:
maximum conversion separation (y-axis)
given minimum qubit separation (x-axis)

Next steps:

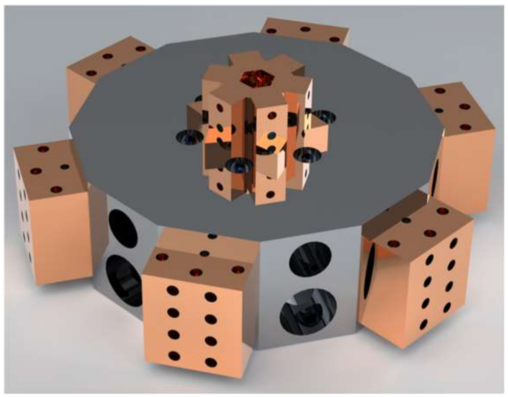
- Constraints for SNAIL-qubit conversion
- Multi-qubit gate + speed limits
- Scheduling pulses for double SNAILS

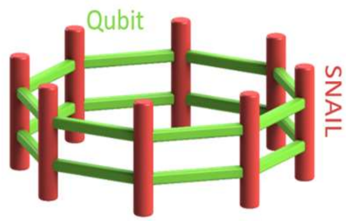
➢ Qubit bandwidth [4,6] GHz

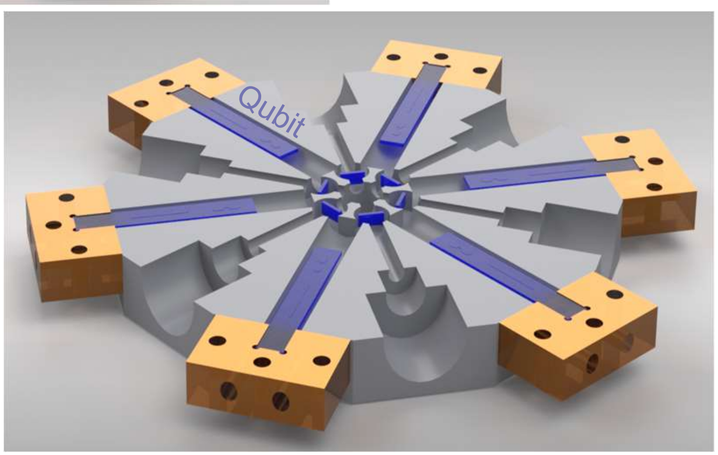
McKinney, et al. **arXiv:2409.18262** (2024)

Physical realization of the module (and potential problems?)

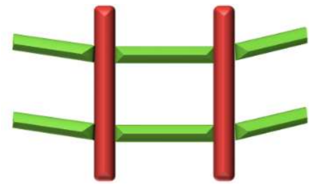
Corral

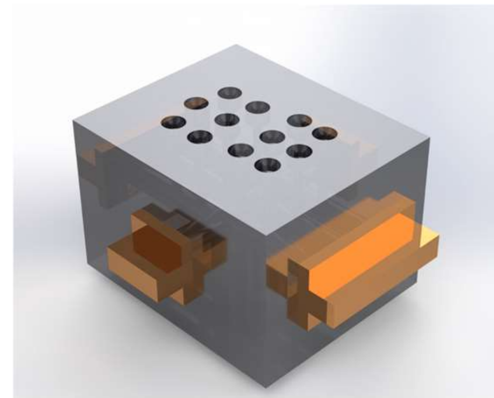


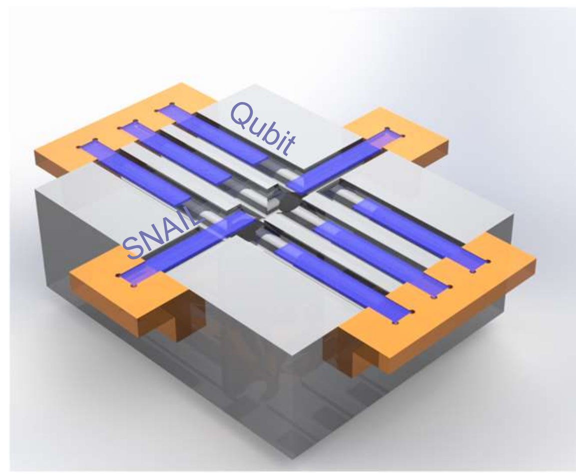




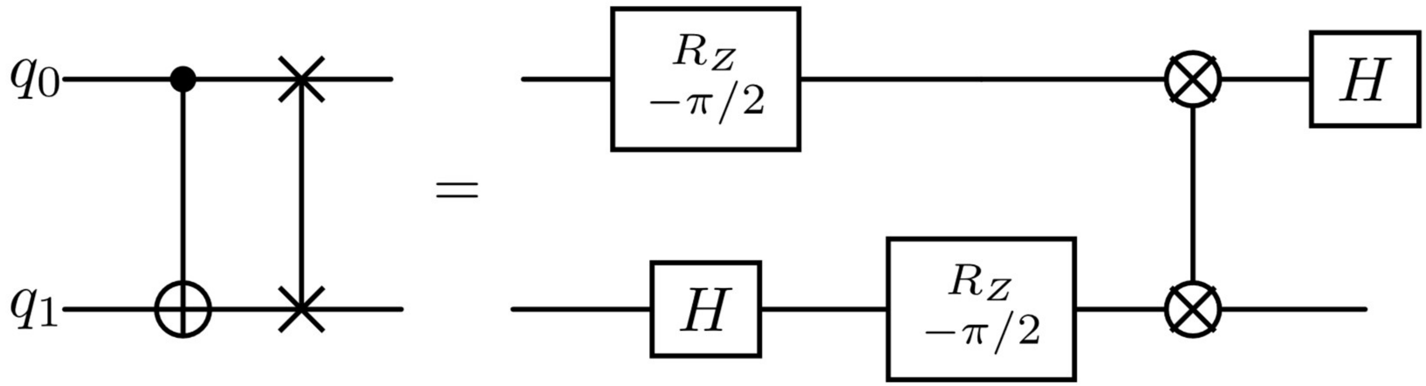
Fence-line

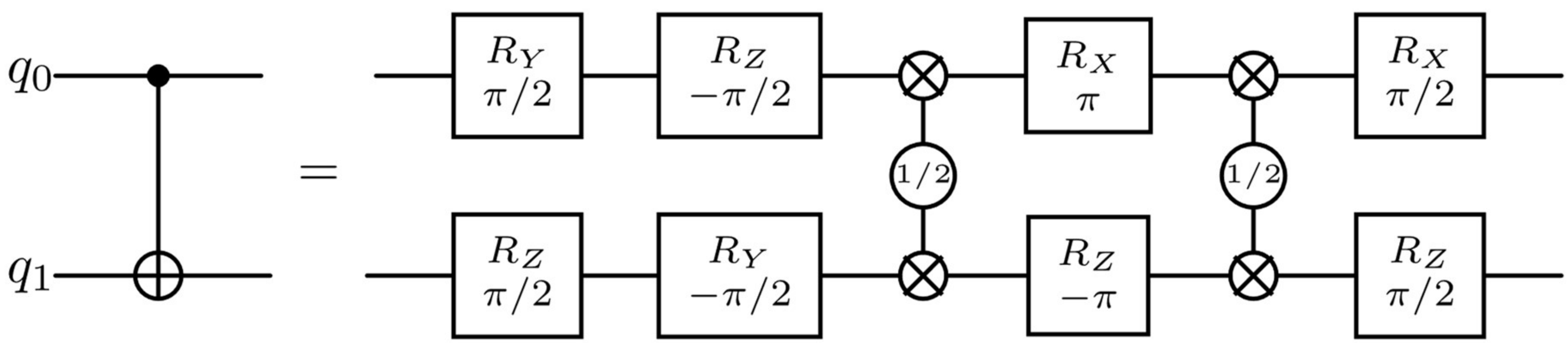






Decomposition identities



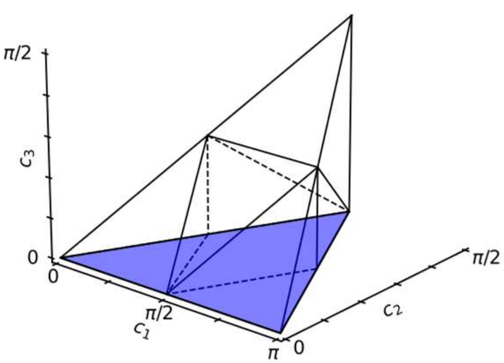


Schuch, et al. **Physical Review A 67.3** (2003)
Huang, et al. **Physical Review Letters** (2023)
McKinney, et al. **HPCA** (2024)

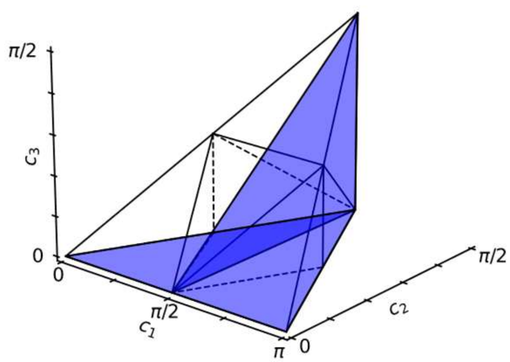
Mirror-inclusive coverage sets

- Compute effective coverage volumes using monodromy polytopes

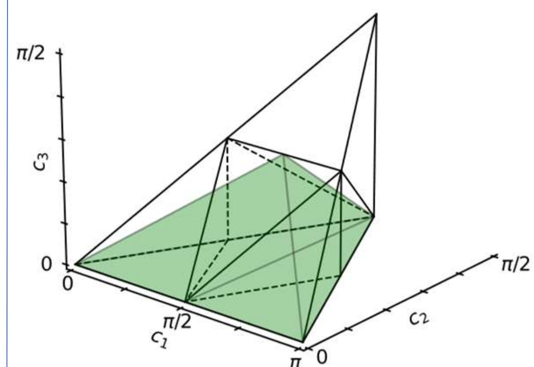
CNOT



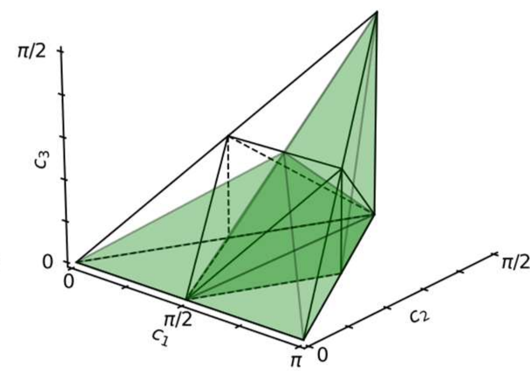
CNOT+Mirrors



$\sqrt{\text{iSWAP}}$



$\sqrt{\text{iSWAP}}$+Mirrors



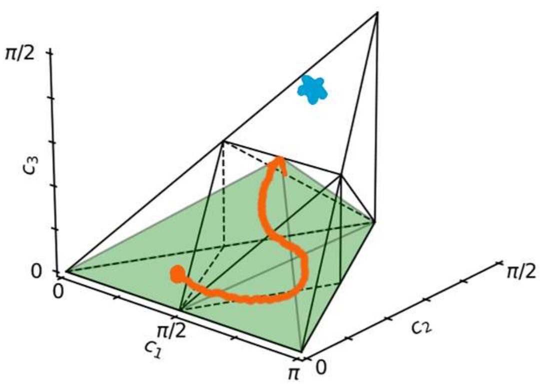
Peterson, et al. **Quantum 4** (2020)
Peterson, et al. **Quantum 6** (2022)
McKinney, et al. **HPCA** (2024)

$k = 2$

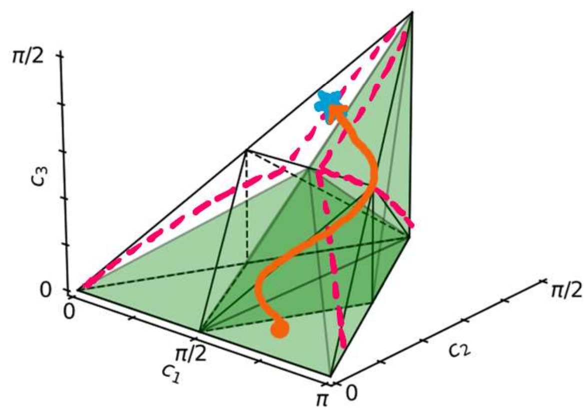
Monte Carlo Haar scores

- Intuition: Approximate decomp threshold defines an acceptable inflated polytope volume.

Javadi, Ali. **APS March Meeting** (2023)



Exact: Fail



Approx. + Mirrors: Success

Lao, et al. **ISCA** (2021)
McKinney, et al. **HPCA** (2024)

Monte Carlo Haar scores

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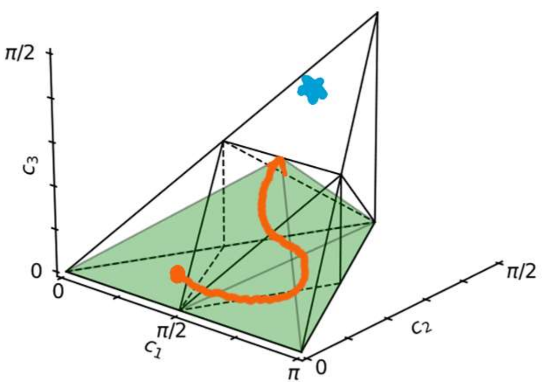
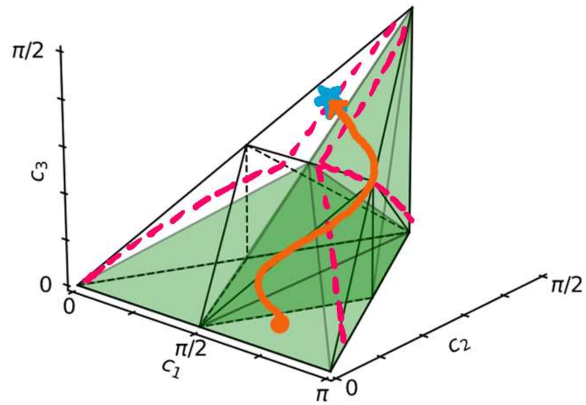
Javadi, Ali. **APS March Meeting** (2023)



Exact: Fail

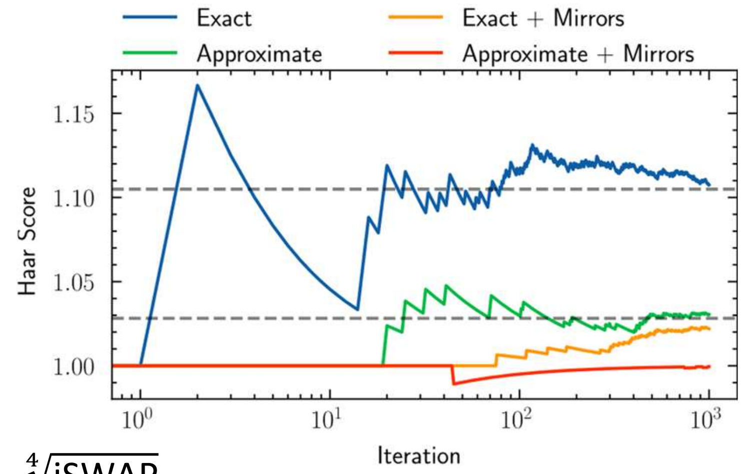


Approx. + Mirrors: Success

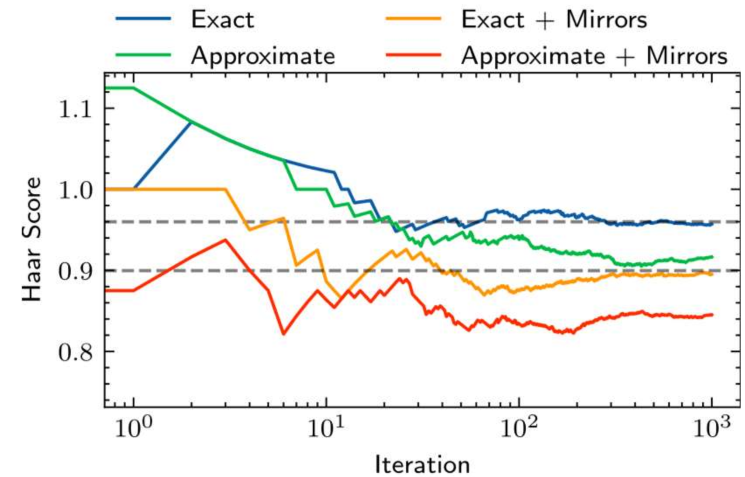
- $\sqrt{\text{iSWAP}}$ with approximate decomp + mirrors has an 8.8% relative decrease in total infidelity

Lao, et al. **ISCA** (2021)
McKinney, et al. **HPCA** (2024)

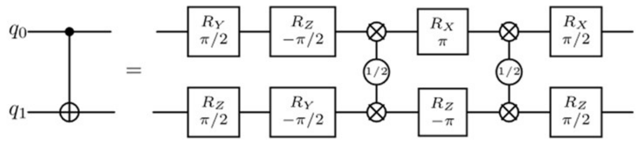
$\sqrt{\text{iSWAP}}$

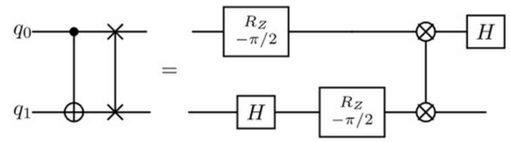


$\sqrt[4]{\text{iSWAP}}$



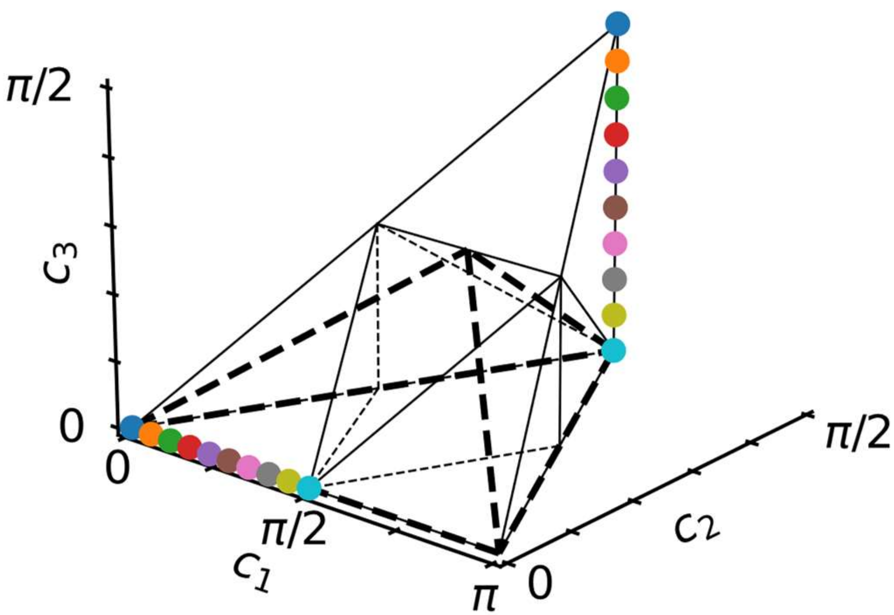
Why does this work?





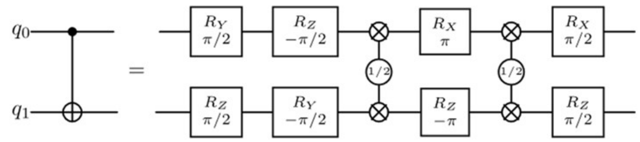
- CPHASE gates mirror to pSWAP gates

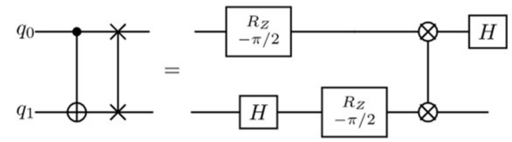
$$(a', b', c') = \begin{cases} (\frac{\pi}{4} + c, \frac{\pi}{4} - b, \frac{\pi}{4} - a) & \text{if } a \leq \frac{\pi}{4} \\ (\frac{\pi}{4} - c, \frac{\pi}{4} - b, a - \frac{\pi}{4}) & \text{else} \end{cases}$$



Peterson, et al. **Quantum 6** (2022)

Why does this work?





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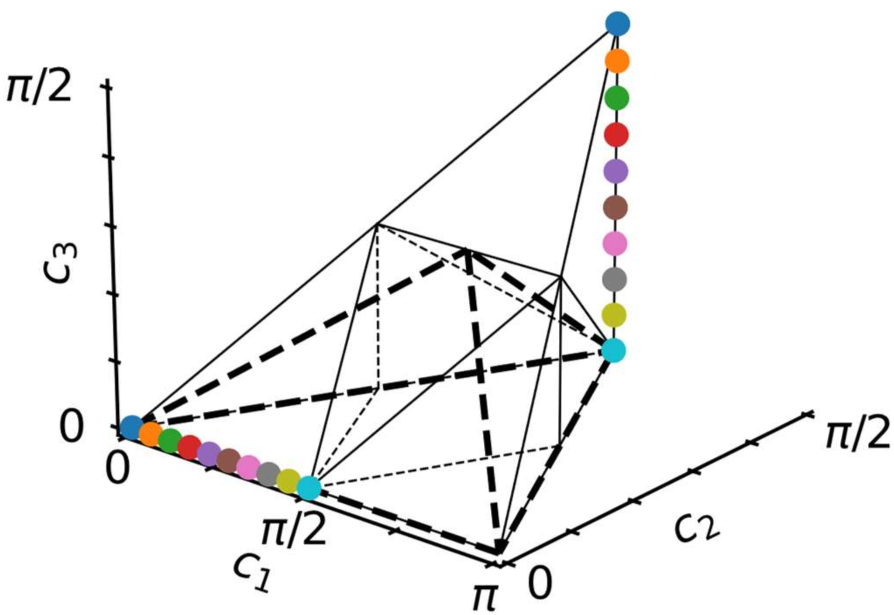
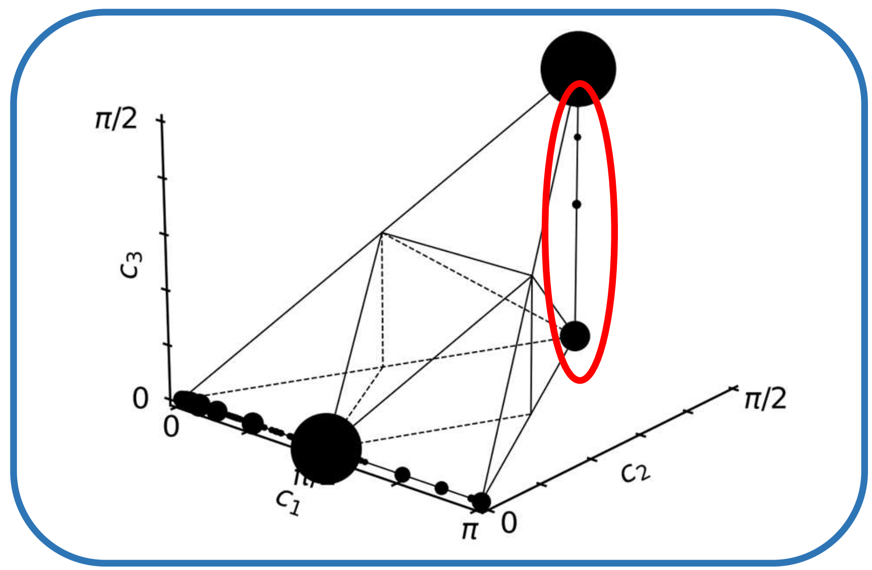
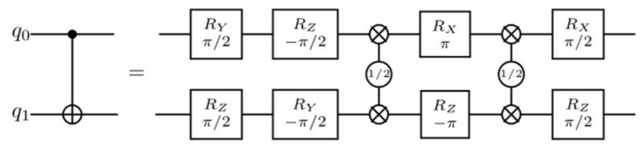
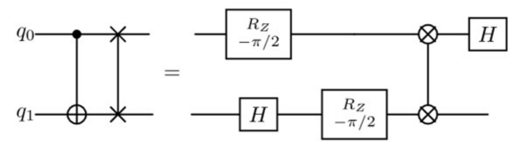




Peterson, et al. **Quantum 6** (2022)

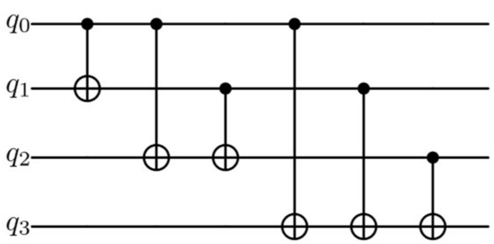
Using mirrors for data movement



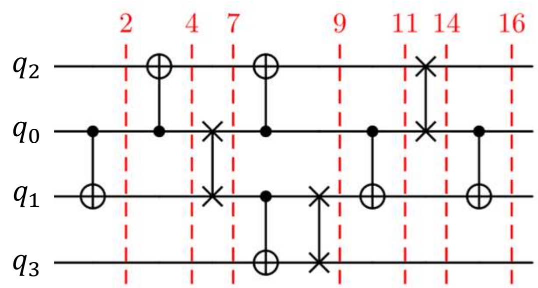


- Intuition: For every CX, decide whether output qubit ordering is (q0, q1) or (q1, q0) based on whether it makes the qubits closer to their next qubit pair

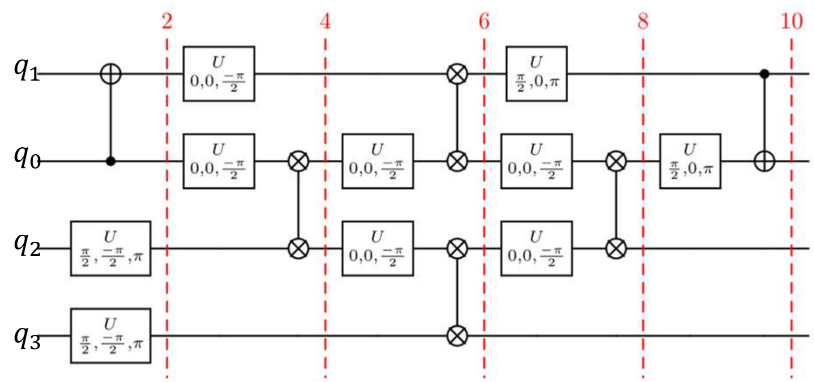
Goal: Full entanglement on a line topology



Qiskit

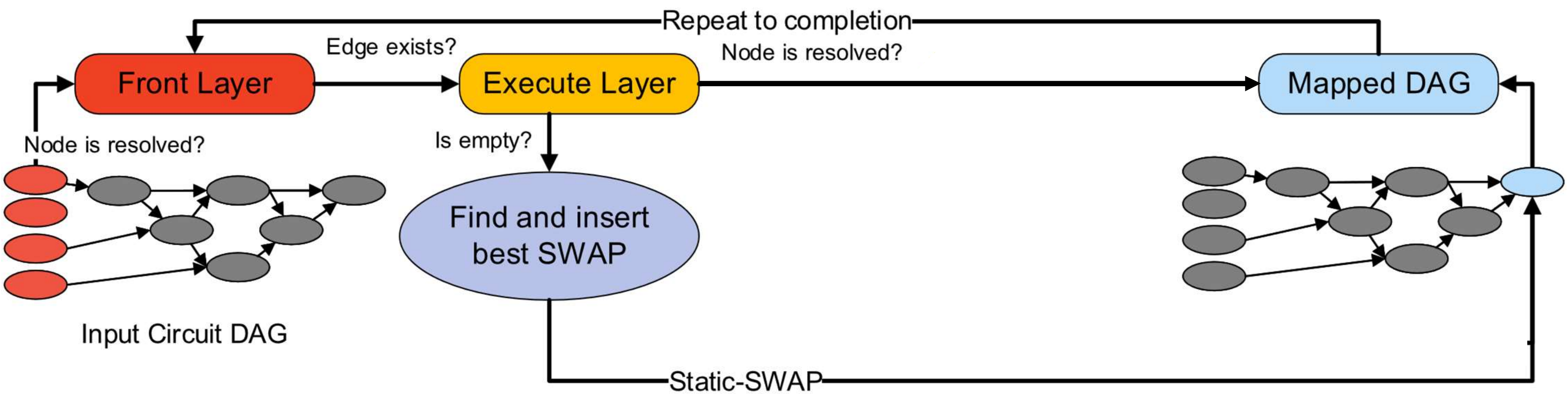


MIRAGE



McKinney, et al. **HPCA** (2024)

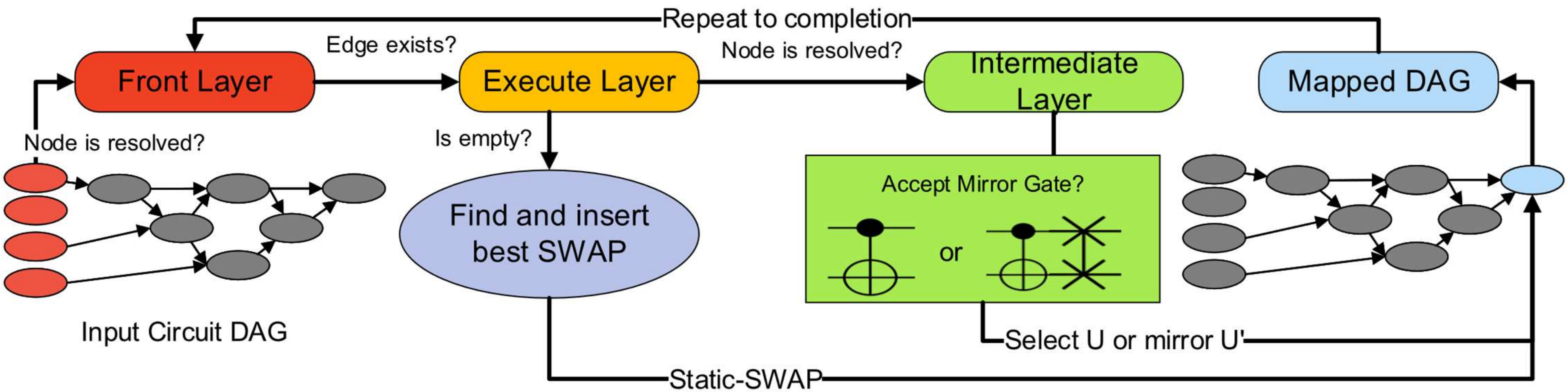
Mirage flow



- Simple yet powerful modification to SABRE:
  - Each gate must pass through an Intermediate Layer
  - Considers if substituting the *mirror* would reduce topological distance cost

Li, et al. **ASPLOS** (2019)

Mirage flow

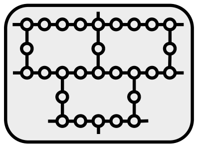


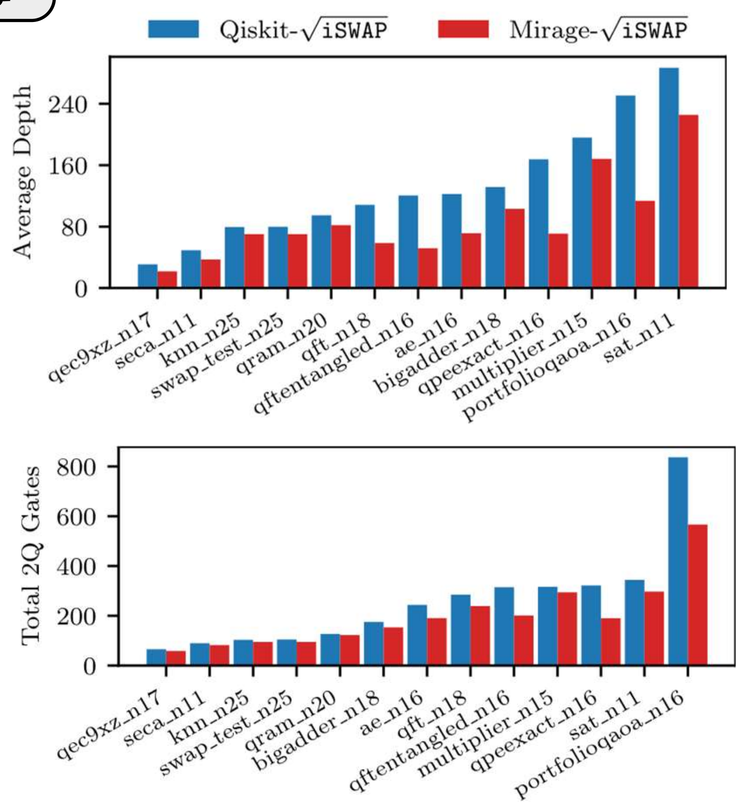
- Simple yet powerful modification to SABRE:
  - Each gate must pass through an Intermediate Layer
  - Considers if substituting the *mirror* would reduce topological distance cost

Li, et al. **ASPLOS** (2019)

Circuit depth reduction

- For the Heavy-Hex topology
  - Average depth decrease of **31.19%**
  - Average total gate decrease of **16.97%**





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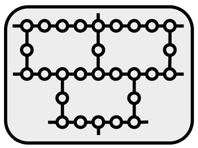
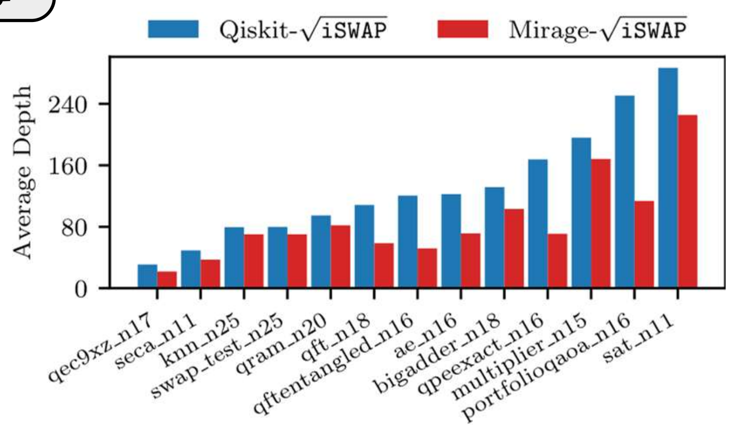
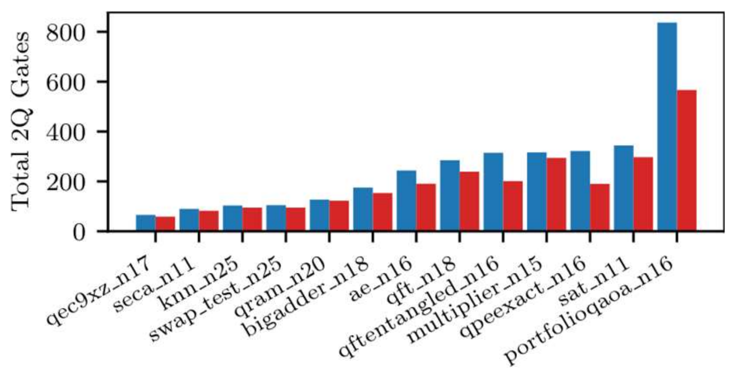






- Qiskit Transpiler Plugin

Usage

```
from qiskit.transpiler import CouplingMap
coupling_map = CouplingMap.from_grid(6, 6)
```

1. Use as a Qiskit-Plugin

Integrate MIRAGE into your existing transpilation pipeline:

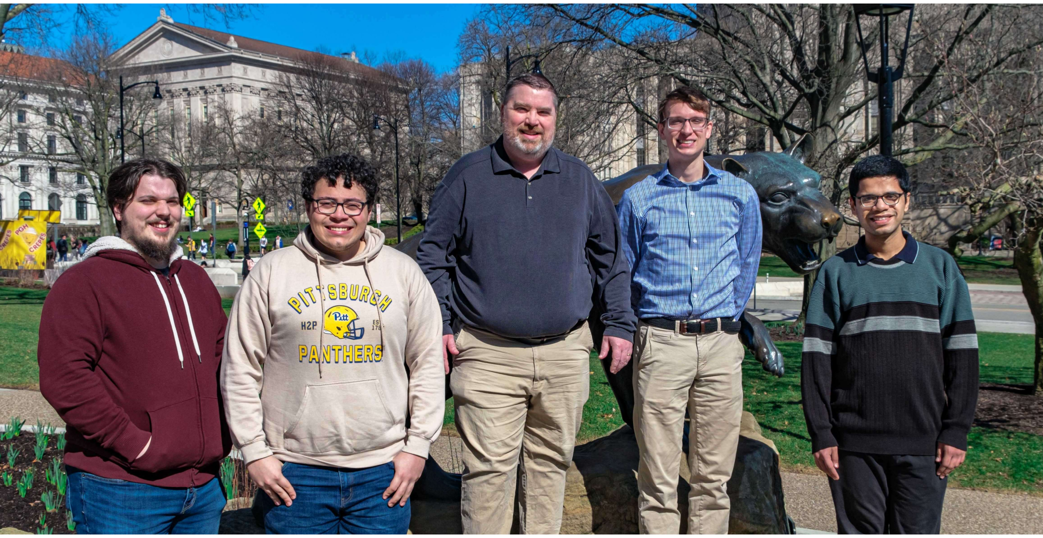
```
from qikist import transpile
mirage_qc = transpile(
            qc, # input circuit
            optimization_level = 3, # default: Qiskit's highest level
            coupling_map=coupling_map,
            basis_gates= ["u", "xx_plus_yy", "id"],
            routing_method="mirage",
            layout_method="sabre_layout_v2",
)
```

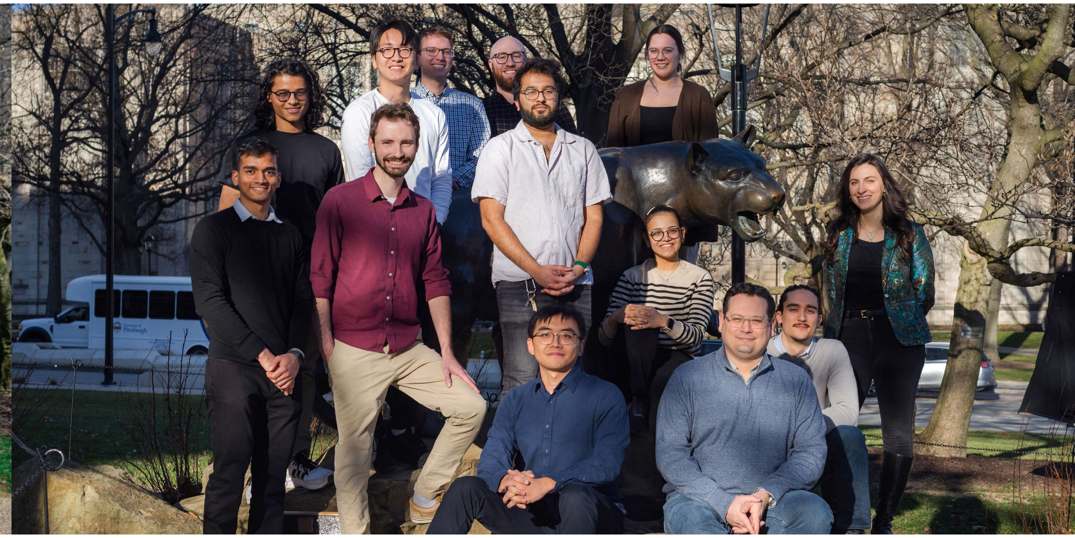
- Software optimizations:
  - Depth post-selection criteria
  - Variable mirror acceptance thresholds
  - Fast block consolidate w/ coord caching

https://github.com/Pitt-JonesLab/mirror-gates

Conclusions

- Evaluate $\sqrt{\text{iSWAP}}$ as a choice basis gate for optimized quantum ISAs
- Qubit frequency allocation over hardware-aware gate errors for SNAIL Corral design
- Significant circuit optimization with MIRAGE, **reducing depth by ~30%**





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